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Please let me for fifteen minutes completely ignore all gas in the cores of galaxies. Clearly the core gas exhibits fascinating, varied and consequential phenomena. Still it would seem to me a plausible assumption that while the gas might be decisively affected by the stars, this gas might not have major effects on the stars, at least in their dynamical behavior. Accordingly I shall here consider purely stellar systems.

Obviously in the core of a galaxy, for dynamical consideration we have to distinguish sharply between the nucleus of a galaxy and the larger components of the galaxy co-occupying the core. Following Baade's usage the nucleus is a stellar system of at most a few parsecs in radius and of a density substantially higher than the central densities of the other components. That the nucleus of a galaxy, at least in its equilibrium state -though not necessarily in its origin- is a distinct, self-contained stellar system seems to me strongly indicated by the rotation velocity measurements in the nucleus of the Andromeda Nebula (Lallemand et al. 1960; Walker, 1974) as well as by the photometric profile determination of the same nucleus (Light et al. 1974). For a nucleus of 10^8 stars the relaxation time for stellar encounters is, according to Spitzer (1971), comparable to a Hubble time while stellar collisions are ignorable. On the other hand, for a nucleus containing 10^{10} stars, which may be the right order of magnitude for more pronounced nuclei, the relaxation time may be comfortably long enough so that encounters may be ignored, but the frequency of collisions is not ignorable. In both cases therefore it would seem that in the centermost portions of nuclei of galaxies either encounters or collisions play a decisive dynamical role but the same may not be true in the outer portions of these nuclei. For the larger subsystems occupying the core of a galaxy, obviously neither encounters nor collisions play a significant role. To stay within the limits of my knowledge, I shall limit myself here to encounterless and collisionless stellar systems, a marginal approximation for the nuclei proper.

Now, why should we worry that any stellar system, including those with which this discussion is concerned, might not have a simple axially

symmetric figure after it is fully relaxed to equilibrium? This worry I think first arose when observations, such as those made by Bertola & Capaccioli (1975) and summarized by Illingworth (1977), showed that elliptical galaxies in general do not rotate as fast as suggested by the then available axially symmetric models for such galaxies. This first alarm, however, was shown not necessarily to indicate more complicated galaxy figures, but to suggest that the observations might be fitted with new axially symmetric models if in these models there were actively involved not only the two classical integrals for such a figure but also a likely existing nonclassical effective integral. Binney (1978) has pointed out that, given the extra integral, axially symmetric models can be constructed with sufficient anisotropy of their internal velocity distribution to obtain the observed oblateness of elliptical galaxies without noticeable help from rotation.

In my mind the strongest, and persisting, reason to worry about possibly more complicated figures of some equilibrium stellar systems is the observed twists. I shall not refer here to the twists observed recently in several photometric studies of elliptical galaxies because these tend to refer to regions in these galaxies well outside the core here considered. I shall rather restrict myself to the Andromeda Nebula. In this neighbor of ours Lindblad (1956) observed that the position angle of the apparent major axis of the bulge deviated by a substantial angle from that of the surrounding disk. He right then pointed out that this phenomenon could be understood by assuming that the bulge was not axially symmetric but had a tri-axial figure with its shortest axis coincident with the axis of the disk but its longest axis oriented relative to the line-of-sight in such a way that the system projected on the sky has an apparent major axis not coincident with that of the disk. More recent observations (Light, et al. 1974) have shown that even the very nucleus of the Andromeda Nebula has an apparent major axis twisted relative to the disk, possibly even a little more than that of the bulge. I would take this as a strong suggestion that even the nucleus of the Andromeda Nebula might, at least in its outer portions, have a tri-axial figure. But in addition I should admit that for me personally probably the strongest stimulus to investigate tri-axial figures is the simple argument that if tri-axial, self-consistent equilibrium configurations exist for stellar systems, nature would have to apply some powerful relaxation mechanism during the formation of a galaxy to avoid them persistently.

What then have theoreticians to say about the existence and characteristics of tri-axial equilibrium configurations? Let us start with the available exact analytical solutions. Freeman (1965) succeeded early in finding such solutions for a tri-axial figure of uniform density. An equally exact analytical solution has recently been derived by Vandervoort (1979) for polytropic models of low index, i.e., models which still have very low internal density concentrations. As for numerically constructed models, the most extensive N-body calculations (Aarseth & Binney 1978; Miller & Smith 1978) seem to have reached fairly secure equilibrium configurations, some of which show a strongly

tri-axial character. These models derived by N-body calculations contain already substantial central density concentrations. Finally, may I mention a tri-axial model I have recently derived with the help of another numerical procedure involving linear programming (Schwarzschild 1979). This model has the density distribution of the Hubble profile which fits the cores of elliptical galaxies. This enumeration of tri-axial theoretical models may be reasonably complete as far as three-dimensional self-consistent models now available is concerned. It is, however, brutally incomplete in that it ignores the extensive and entirely essential efforts of developing a more thorough understanding of the main families of orbits possible in two- and three-dimensional potentials without axial symmetry.

How well do these theoretical models reproduce the observed features of stellar systems? All the models quoted have been arranged to give approximately elliptical isophotes in their projection onto the sky. Thus in form they are made to simulate observed systems. Regarding the density concentration towards the center, the numerically constructed models fit the observations at least in a rough sense, while the exact analytical solutions presently available are too nearly uniform in density. However, I would guess that this latter discrepancy is caused by the mathematical difficulty of the problem rather than by any basic constraints. Thus the photometric comparison -except for the matter of twists which I have already discussed- does not seem to tell us much at present. Accordingly the burden of critical comparison should fall on the velocity characteristics, particularly the rotational motions within the models. However, exactly with regard to the importance and character of rotation the small number of tri-axial theoretical models presently available could hardly differ more from each other than they do. In Freeman's classical model the majority of stellar orbits are retrograde such that the net mean motion of the stars simulates a rotation slower than the rotation of the figure. In Vandervoort's new polytropic models the stars have no mean stream motion within the frame of the figure so that the entire model displays the solid body rotation of the figure itself. The particular numerical model studied by Miller & Smith in greatest detail shows stellar stream motions within the figure in a prograde sense such that the net observed rotational motion of the stars is greater than that of the figure -a situation which I believe many of us would offhand consider the most likely one. Finally, the one numerical model I have recently constructed has no figure rotation (a matter of initial choice) and no stream motion simulating a rotation around its shortest axis (again basically a matter of choice though it is less clear that another choice would have permitted a solution). Thus the available models run the full gamut. At the one end, Vandervoort's model family has isotropic velocity dispersions such that the tri-axial figure is entirely caused by the rotation. At the other end, my specific model has zero rotation around the short axis such that the tri-axial figure must be entirely caused by an isotropy of the velocity dispersions, much in the way in which Binney suggested axially symmetrical oblate models with very slow or no rotation.

Under these circumstances, if the question of the existence of tri-axial stellar systems -both in theory and in nature- is deemed worth further major efforts, then we clearly need more theoretical models and particularly more insight into the causes of the dynamical differences of such models. On the other hand we equally obviously need more detailed velocity observations for the inner portions of galaxies coupled with the relevant photometric data.

In spite of the clearly highly spotty state of our knowledge and understanding of stellar equilibrium configurations which deviate significantly from axial symmetry I feel even now fairly convinced -and in that I am clearly not alone- that theoretical models for self-consistent tri-axial equilibrium configurations exist and that the cores of some galaxies, such as specifically the Andromeda Nebula, represent such figures.

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