

The book will also be welcomed by revisionists. This brings me to a perceived aspect of this otherwise quite impressive work that I did not like: I have the distinct impression that the underlying theme pursued by the authors is that the Chinese generally did it first and where this is not the case they still did it better. Certainly credit should go where it is due and if research shows that results have been wrongly attributed we must be prepared to revise our views on the historical development of our subject. However, I think that revisionists must present their claims with a degree of caution, otherwise they are in danger of finding themselves guilty of those very offences of which they accuse others. For example, I wonder if the authors can really justify absolute statements such as the following: 'In doing this Jia Xian was the first author to record the triangle of binomial coefficients . . . ' (p. 178); 'However, no other advance in solving linear equations can be found in the work of the Indians from Aryabhata, Bramagupta and Mahavira up to Bhaskara' (p. 387). Inevitably, the Greeks come in for some adverse criticism. I find several of the attempts to appropriate some of their credit rather petty; on a number of occasions we are presented with a proof by a Chinese mathematician writing several centuries later and are told of the elegance, simplicity, clarity or superiority of the Chinese proof as compared to the Greek version (see, for example, pp. 195, 234, 240, 277, 419, 464, 470). So what?

Here is a list of some of the topics which the authors appear to claim as Chinese discoveries or as ideas to which the Chinese made superior contributions: the Euclidean algorithm (p. 3); the rules of arithmetic (including the use of negative numbers) (pp. 36, 388); the rule of three (p. 136); the rules of proportion (including compound ratios) (p. 173); the method of exhaustion (p. 103); Romberg extrapolation (p. 117); the numerical solution of polynomial equations (p. 176); Pascal's triangle (pp. 178, 226); Horner's method (p. 184); Cavalieri's principle (pp. 234, 240); Legendre's formula for the volume of a pentahedron (pp. 254, 287); limits (p. 277); the rule of double false position (p. 354); Gaussian elimination (p. 388); Pythagoras's theorem (pp. 439, 458). These claims certainly have to be given serious consideration.

I. TWEDDLE

SWATERS, G. E. *Introduction to Hamiltonian fluid dynamics and stability theory* (Monographs and Surveys in Pure and Applied Mathematics, Chapman & Hall/CRC, 2000), 274 pp., 1 584 88023 6, £54.95.

Classical mechanics can be firmly grounded on a Hamiltonian and/or Lagrangian formulation. While both approaches are essentially equivalent, Hamiltonian dynamics and the notion of symplecticity have perhaps become the prevalent foundation of mechanics. The extension of the Hamiltonian approach to infinite-dimensional systems, such as wave and fluid dynamics, has become an active area of research over the last twenty to thirty years. Even today the question of which formulation, Hamiltonian or Lagrangian, is to be preferred is largely open. However, it is without doubt that Hamiltonian dynamics has had an important impact on ideal fluid and wave dynamics. This is in particular true for geophysical fluid dynamics, as can be seen from the work of Holm, McIntyre, Morrison, Salmon, Shepherd and others.

The book under review summarizes some of the recent work on Hamiltonian fluid dynamics. In particular, it provides a rather non-technical and entertaining introduction to the Hamiltonian formulation of ideal two-dimensional fluids and stability results for steady flows and travelling waves.

Let me highlight a few of the topics covered in the book. Chapter 2 gives a very compact introduction to the basic concepts in Hamiltonian classical mechanics. The material is self-contained and is kept to the basics. Chapter 3 is concerned with the Hamiltonian structure of two-dimensional ideal incompressible fluids. In a first step the non-canonical Hamiltonian structure of the vorticity formulation is stated and the various properties of the associated Euler–Poisson

bracket are verified. Next this bracket is derived via explicit reduction from a Lagrangian particle formulation of fluid dynamics. The Euler–Poisson bracket leads naturally to the conservation of vorticity in terms of Casimir functionals. The chapter ends with an application of Noether’s Theorem. Unfortunately, the author decided not to mention the concept of particle relabelling, which is at the very heart of the Lagrangian to Euler reduction process. The next chapter provides an extensive discussion of stability results for steady Euler flows. The stability theory of steady flows is complicated by the fact that stationary flows do not, in general, satisfy the first order necessary conditions for an energy minimum. Thus the classical stability methods break down. V. I. Arnold suggested the construction of an invariant pseudo-energy functional. For parallel shear flows Arnold’s linear stability theorems reduce to Fjortoft’s results. Furthermore, Arnold established sufficient conditions which would establish nonlinear stability. The author presents Arnold’s stability results as well as important recent developments, such as Andrew’s Theorem, from a general variational point of view and its associated Hamiltonian formulation.

An interesting generalization of the two-dimensional vorticity equation is provided by the Charney–Hasegawa–Mima (CHM) equation, which arises from the shallow-water equations for rotating fluids in the limit of small Rossby numbers. The CHM equation has dispersive linear wave solutions, called Rossby waves, and has also nonlinear steadily travelling dipole vortex solutions. These solutions play an important role in large scale evolution of the planetary atmosphere. The Hamiltonian structure of the CHM equations and its derivation are discussed in Chapter 5. A large portion of that chapter is then devoted to the stability of steady solutions. An important new feature is the existence of steadily travelling waves. The discussion of their stability leads to important modifications in the previously presented framework; these are also discussed in Chapter 5. The final chapter is concerned with the Hamiltonian structure and the associated stability theory for the celebrated Korteweg–de Vries (KdV) equations.

The book is presented in a refreshingly non-technical style with plenty of details and exercises provided. The reader should be familiar with basic fluid dynamics, classical mechanics, variational calculus, and stability theory. The text can be recommended for advanced undergraduate students and graduate students in applied mathematics and physical sciences. All in all, this is a well-written introduction to Hamiltonian fluid dynamics and basic stability results.

S. REICH

BALSER, W. *Formal power series and linear systems of meromorphic ordinary differential equations* (Universitext, Springer, 2000), xviii+299 pp., 0 387 98690 1 (hardcover), £32.50.

Divergent series occur in a variety of situations. A typical example (considered in the Introduction of the book under review) is of the (meromorphic) ODE,

$$t^2 f' = f - t, \quad (1)$$

which admits the formal asymptotic expansion,

$$f(t) \sim \sum_{n=0}^{\infty} n! t^{n+1},$$

which of course has zero radius of convergence. Attempts to sum divergent series go back at least to Euler. Some such attempts are completely formal and resemble typical student standards of rigour. For example, consider the series,

$$1 - 2 + 4 - 8 + 16 - \dots, \quad (2)$$

the partial sums S_n of which satisfy $S_n = (1 - (-2)^n)/3$. If we want to associate a ‘sum’ S with this series, we see that it has to satisfy $2S = 1 - S$, i.e. $S = 1/3$, which should be inspected