2
Heavy quarks

The light u, d, and s quarks have masses m_q that are small compared to the scale of nonperturbative strong dynamics. Consequently, it is a good approximation to take the $m_q \to 0$ limit of QCD. In this limit QCD has an $SU(3)_L \times SU(3)_R$ chiral symmetry, which can be used to predict some properties of hadrons containing these light quarks. For quarks with masses m_Q that are large compared with the scale of nonperturbative strong dynamics, it is a good approximation to take the $m_Q \to \infty$ limit of QCD. In this limit QCD has spin-flavor heavy quark symmetry, which has important implications for the properties of hadrons containing a single heavy quark.

2.1 Introduction

The QCD Lagrangian in Eq. (1.82) describes the strong interactions of light quarks and gluons. As discussed in Sec. 1.4, there is a nonperturbative scale Λ_{QCD} that is dynamically generated by QCD. A color singlet state, such as a meson made up of a quark–antiquark pair, is bound by the nonperturbative gluon dynamics. If the quarks are light, the typical size of such a system is of the order of $\Lambda_{\text{QCD}}^{-1}$. Consider a $Q\bar{q}$ meson that contains a heavy quark with mass $m_Q \gg \Lambda_{\text{QCD}}$, and a light quark with mass $m_q \ll \Lambda_{\text{QCD}}$. Such a heavy-light meson also has a typical size of the order of $\Lambda_{\text{QCD}}^{-1}$, as for mesons containing only light quarks. The typical momentum transfer between the heavy and light quarks in the $Q\bar{q}$ meson arising from nonperturbative QCD dynamics is of the order of Λ_{QCD}. An important consequence of this fact is that the velocity v of the heavy quark is almost unchanged by such strong interaction effects, even though the momentum p of the heavy quark changes by an amount of the order of Λ_{QCD}, since $\Delta v = \Delta p / m_Q$. A similar argument holds for any hadron containing a single heavy quark Q.

In the limit $m_Q \to \infty$, the heavy quark in the meson can be labeled by a velocity four-vector v that does not change with time. The heavy quark behaves like a static

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external source that transforms as a color triplet, and the meson dynamics reduces to that of light degrees of freedom interacting with this color source. One sees immediately that the mass of the heavy quark is completely irrelevant in the limit $m_Q \to \infty$, so that all heavy quarks interact in the same way within heavy mesons. This leads to *heavy quark flavor symmetry:* the dynamics is unchanged under the exchange of heavy quark flavors. The $1/m_Q$ corrections take into account finite mass effects and differ for quarks of different masses. As a result, heavy quark flavor symmetry breaking effects are proportional to $(1/m_{Q_i} - 1/m_{Q_j})$, where Q_i and Q_j are any two heavy quark flavors. The only strong interaction of a heavy quark is with gluons, as there are no quark–quark interactions in the Lagrangian. In the $m_Q \to \infty$ limit, the static heavy quark can only interact with gluons via its chromoelectric charge. This interaction is spin independent. This leads to *heavy quark spin symmetry:* the dynamics is unchanged under arbitrary transformations on the spin of the heavy quark. The spin-dependent interactions are proportional to the chromomagnetic moment of the quark, and so are of the order of $1/m_Q$. Heavy quark spin symmetry breaking does not have to be proportional to the difference of $1/m_Q$'s, since the spin symmetry is broken even if there are two heavy quarks with the same mass. The heavy quark $SU(2)$ spin symmetry and $U(N_h)$ flavor symmetries (for N_h heavy flavors) can be embedded into a larger $U(2N_h)$ spin-flavor symmetry in the $m_Q \to \infty$ limit. Under this symmetry the $2N_h$ states of the N_h heavy quarks with spin up and down transform as the fundamental representation. We will see in Sec. 2.6 that the effective Lagrangian can be written in a way that makes this symmetry manifest.

2.2 Quantum numbers

Heavy hadrons contain a heavy quark as well as light quarks and/or antiquarks and gluons. All the degrees of freedom other than the heavy quark are referred to as the light degrees of freedom ℓ. For example, a heavy $Q\bar{q}$ meson has an antiquark $\bar{q}$, gluons, and an arbitrary number of $\bar{q}q$ pairs as the light degrees of freedom. Although the light degrees of freedom are some complicated mixture of the antiquark $\bar{q}$, gluons, and $\bar{q}q$ pairs, they must have the quantum numbers of a single antiquark $\bar{q}$. The total angular momentum of the hadron $\mathbf{J}$ is a conserved operator. We have also seen that the spin of the heavy quark $\mathbf{S}_Q$ is conserved in the $m_Q \to \infty$ limit. Therefore, the spin of the light degrees of freedom $\mathbf{S}_\ell$ defined by

$$\mathbf{S}_\ell \equiv \mathbf{J} - \mathbf{S}_Q \tag{2.1}$$

is also conserved in the heavy quark limit. The light degrees of freedom in a hadron are quite complicated and include superpositions of states with different particle numbers. Nevertheless, the total spin of the light degrees of freedom is a good quantum number in heavy hadrons. We will define the quantum numbers j,

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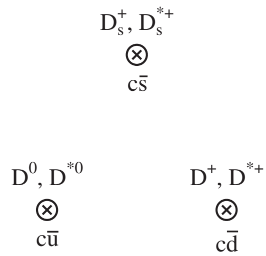

Fig. 2.1. Flavor $SU(3)$ weight diagram for the spin-0 pseudoscalar and spin-1 vector $c\bar{q}$ mesons. The corresponding $b\bar{q}$ mesons are the $\bar{B}_s^0$, $\bar{B}^-$, and $\bar{B}^0$, and their spin-1 partners. The vertical direction is hypercharge, and the horizontal direction is I_3, the third component of isospin.

s_Q, and s_ℓ as the eigenvalues $\mathbf{J}^2 = j\,(j+1)$, $\mathbf{S}_Q^2 = s_Q(s_Q+1)$, and $\mathbf{S}_\ell^2 = s_\ell\,(s_\ell+1)$ in the state $H^{(Q)}$. Heavy hadrons come in doublets (unless $s_\ell = 0$) containing states with total spin $j_\pm = s_\ell \pm 1/2$ obtained by combining the spin of the light degrees of freedom with the spin of the heavy quark $s_Q = 1/2$. These doublets are degenerate in the $m_Q \to \infty$ limit. If $s_\ell = 0$, there is only a single $j = 1/2$ hadron.

Mesons containing a heavy quark Q are made up of a heavy quark and a light antiquark $\bar{q}$ (plus gluons and $q\bar{q}$ pairs). The ground state mesons are composed of a heavy quark with $s_Q = 1/2$ and light degrees of freedom with $s_\ell = 1/2$, forming a multiplet of hadrons with spin $j = 1/2 \otimes 1/2 = 0 \oplus 1$ and negative parity, since quarks and antiquarks have opposite intrinsic parity. These states are the D and D^* mesons if Q is a charm quark, and the $\bar{B}$ and $\bar{B}^*$ mesons if Q is a b quark. The field operators which annihilate these heavy quark mesons with velocity v are denoted by $P_v^{(Q)}$ and $P_{v\mu}^{*(Q)}$, respectively. The light antiquark can be either a $\bar{u}$, $\bar{d}$, or $\bar{s}$ quark, so each of these heavy meson fields form a $\bar{\mathbf{3}}$ representation of the light quark flavor group $SU(3)_V$. The $SU(3)$ weight diagram for the $\bar{\mathbf{3}}$ mesons is shown in Fig. 2.1.

In the nonrelativistic constituent quark model, the first excited heavy meson states have a unit of orbital angular momentum between the constituent antiquark and the heavy quark. These $L = 1$ mesons have $s_\ell = 1/2$ or $3/2$, depending on how the orbital angular momentum is combined with the antiquark spin. The $s_\ell = 1/2$ mesons form multiplets of spin parity 0^+ and 1^+ states named (for $Q = c$) D_0^* and D_1^*, and the $s_\ell = 3/2$ mesons form multiplets of 1^+ and 2^+ states named (for $Q = c$) D_1 and D_2^*. Properties of the $s_\ell = 1/2$ and $s_\ell = 3/2$ states are related in the nonrelativistic constituent quark model, but not by heavy quark symmetry.

Baryons containing a heavy quark consist of a heavy quark and two light quarks, plus gluons and $q\bar{q}$ pairs. The lowest-lying baryons have $s_\ell = 0$ and $s_\ell = 1$ and form $\bar{\mathbf{3}}$ and $\mathbf{6}$ representations of $SU(3)_V$, which are shown in Figs. 2.2 and 2.3, respectively. We can easily understand this pattern in the nonrelativistic

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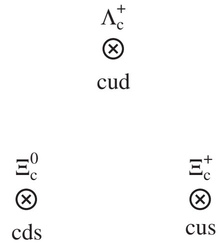

Fig. 2.2. Flavor $SU(3)$ weight diagram for the $\bar{\mathbf{3}}$ spin-$1/2$ $c\,[qq]$ baryons. The corresponding $b\,[qq]$ baryons are the Λ_b^0, Ξ_b^- and Ξ_b^0. The vertical direction is hypercharge, and the horizontal direction is I_3, the third component of isospin.

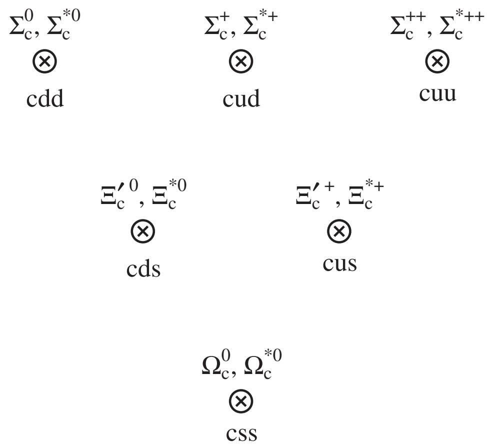

Fig. 2.3. Flavor $SU(3)$ weight diagram for the $\mathbf{6}$ spin-$1/2$ and spin-$3/2$ $c\,(qq)$ baryons. The corresponding $b\,(qq)$ baryons are the spin-$1/2$ $\Sigma_b^{-,0,+}$, $\Xi_b'^{-,0}$, and Ω_b^-, and their spin-$3/2$ partners. The vertical direction is hypercharge, and the horizontal direction is I_3, the third component of isospin.

constituent quark model. In this model the ground-state baryons have no orbital angular momentum and the spatial wave function for the two light constituent quarks is symmetric under their interchange. The wave function is also completely antisymmetric in color. Fermi statistics then demands that for $s_\ell = 0$, where the spin wave function is antisymmetric, the $SU(3)_V$ flavor wave function is also antisymmetric, and hence transforms as $(\mathbf{3} \times \mathbf{3})_{\text{antisymmetric}} = \bar{\mathbf{3}}$. For $s_\ell = 1$, the $SU(3)_V$ flavor wave function is symmetric and hence transforms as $(\mathbf{3} \times \mathbf{3})_{\text{symmetric}} = \mathbf{6}$. The $s_\ell = 0$ ground-state baryons have positive parity and total spin of $1/2$, and the spinor fields that destroy these states are denoted by $\Lambda_v^{(Q)}$. The $s_\ell = 1$ ground-state baryons have positive parity and come in a doublet of states with total spins of $1/2$, and $3/2$. We denote the fields that destroy these states by $\Sigma_v^{(Q)}$ and $\Sigma_{v\mu}^{*(Q)}$, respectively. The spectrum of excited baryons is more

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complicated than in the meson sector. In the nonrelativistic constituent quark model, the $L = 1$ baryons come in two types; states with the unit of orbital angular momentum between the two light quarks, and states with the unit of orbital angular momentum between the light quark pair and the heavy quark. The latter are expected to be lower in mass. The lowest-mass hadrons containing c and b quarks are summarized in Tables 2.1 and 2.2, respectively.

2.3 Strong decays of excited heavy hadrons

In many cases the two members of a doublet with spin of the light degrees of freedom s_ℓ can decay by means of a single pion emission to the two members of another lower-mass doublet with spin of the light degrees of freedom s'_ℓ. The orbital angular momentum of the emitted pion (L, L_z) is restricted by parity, angular momentum conservation, and heavy quark spin symmetry. For a given pion partial wave there are four transition amplitudes that are related by heavy quark spin symmetry, e.g., the four amplitudes for $(D_1, D_2^*) \to (D, D^*) + \pi$. It is an instructive exercise to derive these symmetry relations. The derivation only makes use of the standard formula for the addition of angular momenta in quantum mechanics. The first step is to decompose the total angular momentum of the initial and final heavy hadron states j and j' into the spin of the initial and final heavy quark s_Q and s'_Q, and the spin of the initial and final light degrees of freedom s_ℓ and s'_ℓ. Using the Clebsch-Gordan decomposition of $|j, j_z\rangle$ into $|\frac{1}{2}, s_{Q_z}\rangle$ and $|s_\ell, s_{\ell_z}\rangle$,

$$|j, j_z\rangle = \sum_{s_{Q_z}, s_{\ell_z}} \left\langle \tfrac{1}{2}, s_{Q_z}; s_\ell, s_{\ell_z} \middle| \, j, j_z \right\rangle \left| \tfrac{1}{2}, s_{Q_z} \right\rangle \left| s_\ell, s_{\ell_z} \right\rangle, \tag{2.2}$$

and the corresponding decomposition of $|j', j'_z\rangle$ into $|\frac{1}{2}, s'_{Q_z}\rangle$ and $|s'_\ell, s'_{\ell_z}\rangle$, the transition amplitude can be written in the form

$$\begin{aligned}
&\mathcal{M}\left[H^{(Q)}(j, j_z) \to H^{(Q)}(j', j'_z) + \pi(L, L_z)\right] \\
&\quad = \langle \pi\,(L, L_z);\; j', j'_z | H_{\text{eff}} | j, j_z \rangle \\
&\quad = \sum \left\langle \pi\,(L, L_z);\; \tfrac{1}{2}, s'_{Qz};\; s'_\ell, s'_{\ell z} \middle| \, H_{\text{eff}} \, \middle| \tfrac{1}{2}, s_{Qz};\; s_\ell, s_{\ell z} \right\rangle \\
&\qquad \times \left\langle \tfrac{1}{2}, s'_{Qz}; s'_\ell, s'_{\ell z} \middle| \, j', j'_z \right\rangle \left\langle \tfrac{1}{2}, s_{Qz}; s_\ell, s_{\ell z} \middle| \, j, j_z \right\rangle.
\end{aligned} \tag{2.3}$$

Eq. (2.3) is schematic and only keeps track of the group theory factors. The effective strong interaction Hamiltonian, H_{eff}, conserves the spin of the heavy quark and of the light degrees of freedom separately. The Wigner-Eckart theorem then implies that the hadronic matrix element must have the form

$$\begin{aligned}
&\left\langle \pi\,(L, L_z);\; \tfrac{1}{2}, s'_{Qz};\; s'_\ell, s'_{\ell z} \middle| \, H_{\text{eff}} \, \middle| \tfrac{1}{2}, s_{Qz};\; s_\ell, s_{\ell z} \right\rangle \\
&\qquad = \delta_{s_{Qz}, s'_{Qz}} \langle L, L_z; s'_\ell, s'_{\ell z} | s_\ell, s_{\ell z} \rangle \langle L, s'_\ell \parallel H_{\text{eff}} \parallel s_\ell \rangle,
\end{aligned} \tag{2.4}$$

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Table 2.1. *The lowest-mass hadrons containing a c quark*[a]

Hadron	Mass (MeV)	Quark Content	J^P	s_ℓ
D^+	1869.3 ± 0.5	$c\bar{d}$	0^-	$1/2$
D^{*+}	2010.0 ± 0.5		1^-	
D^0	1864.6 ± 0.5	$c\bar{u}$	0^-	$1/2$
D^{*0}	2006.7 ± 0.5		1^-	
D_s^+	1968.5 ± 0.6	$c\bar{s}$	0^-	$1/2$
D_s^{*+}	2112.4 ± 0.7		1^-	
D_0^*		$c\bar{q}$	0^+	$1/2$
D_1^*	2461 ± 50		1^+	
D_1	2422.2 ± 1.8	$c\bar{q}$	1^+	$3/2$
D_2^*	2458.9 ± 2.0		2^+	
Λ_c^+	2284.9 ± 0.6	$c[ud]$	$1/2^+$	0
Ξ_c^+	2465.6 ± 1.4	$c[us]$	$1/2^+$	0
Ξ_c^0	2470.3 ± 1.8	$c[ds]$	$1/2^+$	0
Σ_c^{++}	2452.8 ± 0.6	$c(uu)$	$1/2^+$	1
Σ_c^{*++}	2519.4 ± 1.5		$3/2^+$	
Σ_c^+	2453.6 ± 0.9	$c(ud)$	$1/2^+$	1
Σ_c^{*+}			$3/2^+$	
Σ_c^0	2452.2 ± 0.6	$c(dd)$	$1/2^+$	1
Σ_c^{*0}	2517.5 ± 1.4		$3/2^+$	
$\Xi_c^{\prime +}$	2573.4 ± 3.3	$c(us)$	$1/2^+$	1
Ξ_c^{*+}	2644.6 ± 2.1		$3/2^+$	
$\Xi_c^{\prime 0}$	2577.3 ± 3.4	$c(ds)$	$1/2^+$	1
Ξ_c^{*0}	2643.8 ± 1.8		$3/2^+$	
Ω_c^0	2704 ± 4	$c(ss)$	$1/2^+$	1
Ω_c^{*0}			$3/2^+$	

[a] Heavy quark spin symmetry multiplets are listed together. For the excited mesons, the masses quoted correspond to $q = u, d$. Excited charm masses with quark content $c\bar{s}$ and excited charm baryons have also been observed.

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Table 2.2. *The lowest-mass hadrons containing a b quark*[a]

Hadron	Mass (MeV)	Quark Content	J^P	s_ℓ
$\bar{B}^0$ $\bar{B}^{*0}$	5279.2 ± 1.8 5324.9 ± 1.8	$b\bar{d}$	0^- 1^-	$1/2$
$\bar{B}^-$ $\bar{B}^{*-}$	5278.9 ± 1.8 5324.9 ± 1.8	$b\bar{u}$	0^- 1^-	$1/2$
$\bar{B}_s^0$ $\bar{B}_s^{*0}$	5369.3 ± 2.0	$b\bar{s}$	0^- 1^-	$1/2$
$\bar{B}_0^*$ $\bar{B}_1^*$		$b\bar{q}$	0^+ 1^+	$1/2$
$\bar{B}_1$ $\bar{B}_2^*$		$b\bar{q}$	1^+ 2^+	$3/2$
Λ_b^0	5624 ± 9	$b[ud]$	$1/2^+$	0
Ξ_b^0		$b[us]$	$1/2^+$	0
Ξ_b^-		$b[ds]$	$1/2^+$	0
Σ_b^+ Σ_b^{*+}		$b(uu)$	$1/2^+$ $3/2^+$	1
Σ_b^0 Σ_b^{*0}		$b(ud)$	$1/2^+$ $3/2^+$	1
Σ_b^- Σ_b^{*-}		$b(dd)$	$1/2^+$ $3/2^+$	1
$\Xi_b^{\prime 0}$ Ξ_b^{*0}		$b(us)$	$1/2^+$ $3/2^+$	1
$\Xi_b^{\prime -}$ Ξ_b^{*-}		$b(ds)$	$1/2^+$ $3/2^+$	1
Ω_b^- Ω_b^{*-}		$b(ss)$	$1/2^+$ $3/2^+$	1

[a] Heavy quark spin symmetry multiplets are listed together.

where the final term is the reduced matrix element. Substituting into Eq. (2.3) yields

$$\begin{aligned}
\mathcal{M} &= \sum \left\langle \tfrac{1}{2}, s_{Qz}; s_\ell, s_{\ell z} \middle| \, j, j_z \right\rangle \langle L, s'_\ell \parallel H_{\rm eff} \parallel s_\ell \rangle \\
&\quad \times \left\langle \tfrac{1}{2}, s_{Qz}; s'_\ell, s'_{\ell z} \middle| \, j', j'_z \right\rangle \left\langle L, L_z; s'_\ell, s'_{\ell z} \middle| \, s_\ell, s_{\ell z} \right\rangle \\
&= (-1)^{L+s'_\ell+\frac{1}{2}+j} \sqrt{(2s_\ell+1)(2j'+1)} \begin{Bmatrix} L & s'_\ell & s_\ell \\ \frac{1}{2} & j & j' \end{Bmatrix} \\
&\quad \times \langle L, (j_z - j'_z); j', j'_z | j, j_z \rangle \langle L, s'_\ell \parallel H_{\rm eff} \parallel s_\ell \rangle,
\end{aligned} \tag{2.5}$$

where we have rewritten the product of Clebsch-Gordan coefficients in terms of $6j$ symbols. The total decay rate for $j \to j'$ is given by

$$\begin{aligned}
\Gamma(j \to j'\pi) &\propto (2s_\ell+1)\frac{2j'+1}{2j+1} \sum_{j_z, j'_z} \left| \begin{Bmatrix} L & s'_\ell & s_\ell \\ \frac{1}{2} & j & j' \end{Bmatrix} \right|^2 |\langle L, (j_z - j'_z); j', j'_z | j, j_z \rangle|^2 \\
&= (2s_\ell+1)(2j'+1) \left| \begin{Bmatrix} L & s'_\ell & s_\ell \\ \frac{1}{2} & j & j' \end{Bmatrix} \right|^2,
\end{aligned} \tag{2.6}$$

where we have dropped terms, such as the reduced matrix element, which are the same for different values of j and j'. Equation (2.6) provides relations between the decay rates of the excited $s_\ell = 3/2$ D_1 and D_2^* mesons to the ground state $s_\ell = 1/2$ D or D^* mesons and a pion. These two multiplets have opposite parity and the pion has negative parity, so the pion must be in an even partial wave with $L = 0$ or 2 by parity and angular momentum conservation. The decays $D_2^* \to D\pi$ and $D_2^* \to D^*\pi$ must occur through the $L = 2$ partial wave, while $D_1 \to D^*\pi$ can occur by either the $L = 0$ or $L = 2$ partial wave. The $L = 0$ partial wave amplitude for $D_1 \to D^*\pi$ vanishes by heavy quark symmetry since

$$\begin{Bmatrix} 0 & 1/2 & 3/2 \\ 1/2 & 1 & 1 \end{Bmatrix} = 0, \tag{2.7}$$

so that all the decays are $L = 2$. Equation (2.6) implies that the $L = 2$ decay rates are in the ratio

$$\begin{array}{ccccccc} \Gamma(D_1 \to D\pi) & : & \Gamma(D_1 \to D^*\pi) & : & \Gamma(D_2^* \to D\pi) & : & \Gamma(D_2^* \to D^*\pi) \\ 0 & : & 1 & : & \frac{2}{5} & : & \frac{3}{5} \end{array}, \tag{2.8}$$

where $\Gamma(D_1 \to D\pi)$ is forbidden by angular momentum and parity conservation. Equation (2.8) holds in the heavy quark symmetry limit, $m_c \to \infty$. There is a very important source of heavy quark spin symmetry violation that is kinematic in origin. For small $\mathbf{p}_\pi$, the decay rates are proportional to $|\mathbf{p}_\pi|^{2L+1}$, which for $L = 2$ is $|\mathbf{p}_\pi|^5$. In the $m_c \to \infty$ limit the D_1 and D_2^* are degenerate and the D and D^* are also degenerate. Consequently this factor does not affect the ratios in Eq. (2.8). However, for the physical value of m_c, the $D^* - D$

mass splitting is ∼140 MeV, which cannot be neglected in comparison with the 450 MeV $D_2^* - D^*$ splitting. Including the factor of $|\mathbf{p}_\pi|^5$, the relative decay rates become

$$\begin{array}{ccccccc} \Gamma(D_1 \to D\pi) & : & \Gamma(D_1 \to D^*\pi) & : & \Gamma(D_2^* \to D\pi) & : & \Gamma(D_2^* \to D^*\pi) \\ 0 & : & 1 & : & 2.3 & : & 0.92 \end{array}. \tag{2.9}$$

As a consequence of Eq. (2.9) we arrive at the prediction $\mathrm{BR}(D_2^* \to D\pi)/\mathrm{BR}(D_2^* \to D^*\pi) \simeq 2.5$, which is in good agreement with the experimental value 2.3 ± 0.6. The prediction for this ratio of branching ratios would have been $2/3$ without including the phase space correction factor.

Phenomenologically, the suppression associated with emission of a low-momentum pion in a partial wave L is $\sim(|\mathbf{p}_\pi|/\Lambda_{\mathrm{CSB}})^{2L+1}$. The fact that the scale $\Lambda_{\mathrm{CSB}} \sim 1$ GeV enables us to understand why the doublet of D_0^* and D_1^* excited $s_\ell = 1/2$ mesons is difficult to observe. For these mesons, heavy quark spin symmetry predicts that their decays to the ground-state doublet by single pion emission occur in the $L = 0$ partial wave. The masses of the $(D_0^*,\ D_1^*)$ are expected to be near the masses of the $(D_1,\ D_2^*)$, and so their widths are larger than those of the D_1 and D_2^* by roughly $(\Lambda_{\mathrm{CSB}}/|\mathbf{p}_\pi|)^4 \sim 20$–40. The D_1 and D_2^* widths are $\Gamma(D_1) = 18.9 \pm 4$ MeV and $\Gamma(D_2^*) = 23 \pm 5$ MeV. Hence the $D_{0,1}^*$ should be broad, with widths greater than 200 MeV, which makes them difficult to observe. The measured width of the D_1^* is 290 ± 100 MeV.

The excited positive parity $s_\ell = 3/2$ mesons D_{s1} and D_{s2}^*, which contain a strange antiquark, have also been observed. The D_{s1} is narrow, $\Gamma(D_{s1}) < 2.3$ MeV, and its decays to D^*K are dominated by the S-wave amplitude. This occurs because the kaon mass is much larger than the pion mass, and so for this decay $|\mathbf{p}_K| \simeq 150$ MeV while in $D_1 \to D^*\pi$ decay $|\mathbf{p}_\pi| \simeq 360$ MeV. Consequently, there is a large kinematic suppression of the D-wave amplitude in $D_{s1} \to D^*K$ decay. The $s_\ell = 1/2$ and $s_\ell = 3/2$ charmed mesons are in a $\bar{\mathbf{3}}$ of $SU(3)_V$, whereas the π, K, and η are in an $\mathbf{8}$. Since there is only one way to combine a $\mathbf{3}$, $\bar{\mathbf{3}}$, and $\mathbf{8}$ into a singlet, $SU(3)_V$ relates the S-wave part of the D_1 decay width to the D_{s1} decay width. Neglecting η final states, which are phase space suppressed, $SU(3)_V$ light quark symmetry leads to the expectation that $\Gamma_{S-\mathrm{wave}}(D_1) \approx (3/4)\Gamma(D_{s1}) \times |\mathbf{p}_\pi|/|\mathbf{p}_K| < 4.1$ MeV.

2.4 Fragmentation to heavy hadrons

A heavy quark produced in a high-energy process will materialize as a hadron containing that heavy quark. Once the "off-shellness" of the fragmenting heavy quark is small compared with its mass, the fragmentation process is constrained by heavy quark symmetry. Heavy quark symmetry implies that the probability, $P^{(H)}_{h_Q \to h_s}$, for a heavy quark Q with spin along the fragmentation axis (i.e.,

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helicity) h_Q to fragment to a hadron H with spin s, spin of the light degrees of freedom s_ℓ, and helicity h_s is

$$P^{(H)}_{h_Q \to h_s} = \sum_{h_\ell} P_{Q\to s_\ell}\, p_{h_\ell} |\langle s_Q, h_Q; s_\ell, h_\ell | s, h_s\rangle|^2, \tag{2.10}$$

where $h_\ell = h_s - h_Q$. In Eq. (2.10) $P_{Q\to s_\ell}$ is the probability for the heavy quark to fragment to a hadron with spin of the light degrees of freedom s_ℓ. This probability is independent of the spin and flavor of the heavy quark but will depend on other quantum numbers needed to specify the hadron H. $P_{Q\to s_\ell}$ has the same value for the two hadrons in the doublet related by heavy quark spin symmetry. p_{h_ℓ} is the conditional probability that the light degrees of freedom have helicity h_ℓ, given that Q fragments to s_ℓ. The probabilistic interpretation of the fragmentation process means that $0 \le p_{h_\ell} \le 1$ and

$$\sum_{h_\ell} p_{h_\ell} = 1. \tag{2.11}$$

Like $P_{Q\to s_\ell}$, p_{h_ℓ} is independent of the spin and flavor of the heavy quark, but can depend on the hadron multiplet. The third factor in Eq. (2.10) is the Clebsch-Gordan probability that the hadron H with helicity h_s contains light degrees of freedom with helicity h_ℓ and a heavy quark with helicity h_Q. Parity invariance of the strong interactions implies that

$$p_{h_\ell} = p_{-h_\ell}, \tag{2.12}$$

since reflection in a plane containing the momentum of the fragmenting quark reverses the helicities but leaves the momentum unchanged. Equations (2.11) and (2.12) imply that the number of independent probabilities p_{h_ℓ} is $s_\ell - 1/2$ for mesons and s_ℓ for baryons. At the hadron level, parity invariance of the strong interactions gives the relation $P^{(H)}_{h_Q\to h_s} = P^{(H)}_{-h_Q\to -h_s}$.

Heavy quark spin symmetry has reduced the number of independent fragmentation probabilities. For the ground-state D and D^* mesons, $s_\ell = 1/2$, so $p_{1/2} = p_{-1/2}$, which must both equal $1/2$, since $p_{1/2} + p_{-1/2} = 1$. This gives the relative fragmentation probabilities for a right-handed charm quark,

$$\begin{array}{ccccccc} P^{(D)}_{1/2\to 0} & : & P^{(D^*)}_{1/2\to 1} & : & P^{(D^*)}_{1/2\to 0} & : & P^{(D^*)}_{1/2\to -1} \\ 1/4 & : & 1/2 & : & 1/4 & : & 0 \end{array}. \tag{2.13}$$

Parity invariance of the strong interactions relates the fragmentation probabilities for a left-handed charm quark to those in Eq. (2.13). Heavy quark spin symmetry implies that a charm quark fragments to a D one-third as often as it fragments to a D^*. This prediction disagrees with the experimental data, which give a larger fragmentation probability for the D, and the discrepancy is due to the $D^* - D$ mass difference. We have already seen that the mass difference has an important

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impact on decays of excited charm mesons to the D and D^* and it is not surprising that the mass difference should influence the fragmentation probabilities as well. The $B^* - B$ mass difference is 50 MeV, which is approximately a factor of 3 smaller than the $D^* - D$ mass difference, so one expects the predictions of exact heavy quark symmetry to work better in this case. Recent experimental data from LEP show that the $B^* : B$ ratio is consistent with the predicted value of 3 : 1.

Charm quark fragmentation to the negative parity $s_\ell = 3/2$ multiplet of excited charmed mesons is characterized by the Falk-Peskin parameter $w_{3/2}$, defined as the conditional probability to fragment to helicities $\pm 3/2$,

$$p_{3/2} = p_{-3/2} = \tfrac{1}{2} w_{3/2}, \qquad p_{1/2} = p_{-1/2} = \tfrac{1}{2}(1 - w_{3/2}). \tag{2.14}$$

The value of $p_{\pm 1/2}$ is determined in terms of $w_{3/2}$ since the total fragmentation probability must be unity. The relative fragmentation probabilities are given by Eq. (2.10):

$$\begin{array}{ccccccccc} P^{(D_1)}_{1/2\to 1} & : & P^{(D_1)}_{1/2\to 0} & : & P^{(D_1)}_{1/2\to -1} & : & P^{(D_2^*)}_{1/2\to 2} & : & \\ \tfrac{1}{8}(1-w_{3/2}) & : & \tfrac{1}{4}(1-w_{3/2}) & : & \tfrac{3}{8}w_{3/2} & : & \tfrac{1}{2}w_{3/2} & : & \\ P^{(D_2^*)}_{1/2\to 1} & : & P^{(D_2^*)}_{1/2\to 0} & : & P^{(D_2^*)}_{1/2\to -1} & : & P^{(D_2^*)}_{1/2\to -2} & & \\ \tfrac{3}{8}(1-w_{3/2}) & : & \tfrac{1}{4}(1-w_{3/2}) & : & \tfrac{1}{8}w_{3/2} & : & 0 & & \end{array}. \tag{2.15}$$

Equation (2.15) predicts that the ratio of D_1 to D_2^* production by charm quark fragmentation is 3/5, independent of $w_{3/2}$. Assuming that the decays of the negative parity $s_\ell = 3/2$ charmed mesons are dominated by $D^{(*)}\pi$ final states, the experimental value of this ratio is close to unity. Experimentally the probability of a heavy quark to fragment to the maximal helicities $\pm 3/2$ is small, i.e., $w_{3/2} < 0.24$.

The validity of Eq. (2.10) depends on a crucial assumption. Spin symmetry violation must be negligible in the masses and decays of excited multiplets that can be produced in the fragmentation process and then decay to the final fragmentation product. The spin symmetry violating $D_1 - D_2^*$ mass difference is comparable with the widths of these states, and the spin symmetry violating $D^* - D$ mass difference plays an important role in their decay rates to D and D^*'s. Consequently we do not expect Eq. (2.13) to hold for those D and D^*'s that arise from decays of a D_1 or D_2^*.

2.5 Covariant representation of fields

We have seen that heavy quark symmetry usually implies a degenerate multiplet of states, such as the B and B^*. It is convenient to have a formalism in which the

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entire multiplet of degenerate states is treated as a single object that transforms linearly under the heavy quark symmetries.

The ground-state $Q\bar{q}$ mesons can be represented by a field $H_v^{(Q)}$ that annihilates the mesons, and transforms as a bilinear under Lorentz transformations,

$$H_{v'}^{(Q)\prime}(x') = D(\Lambda)\, H_v^{(Q)}(x)\, D(\Lambda)^{-1}, \tag{2.16}$$

where

$$v' = \Lambda v, \qquad x' = \Lambda x, \tag{2.17}$$

and $D(\Lambda)$ is the Lorentz transformation matrix for spinors, so that

$$H_v^{(Q)}(x) \to H_v^{(Q)\prime}(x) = D(\Lambda)\, H_{\Lambda^{-1}v}^{(Q)}(\Lambda^{-1}x) D(\Lambda)^{-1}. \tag{2.18}$$

The field $H_v^{(Q)}(x)$ is a linear combination of the pseudoscalar field $P_v^{(Q)}(x)$ and the vector field $P_{v\mu}^{*(Q)}(x)$ that annihilate the $s_\ell = 1/2$ meson multiplet. Vector particles have a polarization vector ϵ_μ, with $\epsilon \cdot \epsilon = -1$, and $v \cdot \epsilon = 0$. The amplitude for $P_{v\mu}^{*(Q)}$ to annihilate a vector particle is ϵ_μ. A simple way to combine the two fields into a single field with the desired transformation properties is to define*

$$H_v^{(Q)} = \frac{1+\not{v}}{2}\left[\not{P}_v^{*(Q)} + i P_v^{(Q)} \gamma_5\right]. \tag{2.19}$$

Equation (2.19) is consistent with $P_v^{(Q)}$ transforming as a pseudoscalar, and $P_{v\mu}^{*(Q)}$ as a vector, since γ_5 and γ^μ convert pseudoscalars and vectors into bispinors. The $\left(1+\not{v}\right)/2$ projector retains only the particle components of the heavy quark Q. The relative sign and phase between the P and P^* terms in Eq. (2.19) is arbitrary, and this depends on the choice of phase between the pseudoscalar and vector meson states. The pseudoscalar is multiplied by γ_5 rather than unity, to be consistent with the parity transformation law

$$H_v^{(Q)}(x) \to \gamma^0 H_{v_P}^{(Q)}(x_P)\, \gamma^0, \tag{2.20}$$

where

$$x_P = (x^0, -\mathbf{x}), \qquad v_P = (v^0, -\mathbf{v}). \tag{2.21}$$

The field $H_v^{(Q)}$ satisfies the constraints

$$\not{v} H_v^{(Q)} = H_v^{(Q)}, \qquad H_v^{(Q)} \not{v} = -H_v^{(Q)}. \tag{2.22}$$

The first of these follows directly from $\not{v}\left(1+\not{v}\right) = \left(1+\not{v}\right)$. The second relation follows by anticommuting $\not{v}$ through $H_v^{(Q)}$, and using $v \cdot P_v^{*(Q)} = 0$, since the polarization of physical spin-one particles satisfies $v \cdot \epsilon = 0$.

* For clarity, the superscript (Q) and/or the subscript v will sometimes be omitted.

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It is convenient to introduce the conjugate field

$$\bar{H}_v^{(Q)} = \gamma^0 H_v^{(Q)\dagger}\gamma^0 = \left[P_{v\mu}^{*(Q)\dagger}\gamma^\mu + iP_v^{(Q)\dagger}\gamma_5\right]\frac{1+\not{v}}{2}, \tag{2.23}$$

which also transforms as a bispinor,

$$\bar{H}_v^{(Q)}(x) \to D(\Lambda)\,\bar{H}_{\Lambda^{-1}v}^{(Q)}(\Lambda^{-1}x)D(\Lambda)^{-1}\,, \tag{2.24}$$

since

$$\gamma^0 D(\Lambda)^\dagger\,\gamma^0 = D(\Lambda)^{-1}\,. \tag{2.25}$$

In the rest frame

$$v = v_r = (1, \mathbf{0}) \tag{2.26}$$

the field $H_{v_r}^{(Q)}$ is

$$H_{v_r}^{(Q)} = \begin{pmatrix} 0 & iP_{v_r}^{(Q)} - \boldsymbol{\sigma}\cdot\mathbf{P}_{v_r}^{*(Q)} \\ 0 & 0 \end{pmatrix}, \tag{2.27}$$

using the Bjorken and Drell convention for γ matrices,

$$\gamma^0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \qquad \boldsymbol{\gamma} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{pmatrix}, \qquad \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{2.28}$$

The indices α and β of the field $[H_{v_r}^{(Q)}]_{\alpha\beta}$ label the spinor indices of the heavy quark Q and the light degrees of freedom, respectively. The field $H_{v_r}^{(Q)}$ transforms as a $(1/2, 1/2)$ representation under $S_Q \otimes S_\ell$. The spin operators $\mathbf{S_Q}$ and $\mathbf{S}_\ell$ for the heavy quark and light degrees of freedom acting on the $H_{v_r}^{(Q)}$ field are

$$\begin{aligned} \left[\mathbf{S_Q}, H_{v_r}^{(Q)}\right] &= \tfrac{1}{2}\boldsymbol{\sigma}_{\mathbf{4\times 4}} H_{v_r}^{(Q)}, \\ \left[\mathbf{S}_\ell, H_{v_r}^{(Q)}\right] &= -\tfrac{1}{2} H_{v_r}^{(Q)}\boldsymbol{\sigma}_{\mathbf{4\times 4}}, \end{aligned} \tag{2.29}$$

where $\sigma_{4\times4}^i = i\epsilon_{ijk}[\gamma^j, \gamma^k]/4$ are the usual Dirac rotation matrices in the spinor representation. Under infinitesimal rotations, one finds (neglecting derivative terms that arise from rotating the spatial dependence of the fields) that

$$\delta H_{v_r}^{(Q)} = i\left[\boldsymbol{\theta}\cdot(\mathbf{S}_Q + \mathbf{S}_\ell), H_{v_r}^{(Q)}\right] = \frac{i}{2}\left[\boldsymbol{\theta}\cdot\boldsymbol{\sigma}_{\mathbf{4\times4}}, H_{v_r}^{(Q)}\right], \tag{2.30}$$

so that

$$\delta P_{v_r}^{(Q)} = 0, \qquad \delta\mathbf{P}_{v_r}^{*(Q)} = \boldsymbol{\theta}\times\mathbf{P}_{v_r}^{*(Q)}, \tag{2.31}$$

which are the transformation rules for a spin-zero and spin-one particle, respectively. The fields $P_v^{(Q)}$ and $P_{v\mu}^{*(Q)}$ mix under S_Q or S_ℓ transformations. Under

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heavy quark spin transformations,

$$\delta H_{v_r}^{(Q)} = i\left[\boldsymbol{\theta}\cdot\mathbf{S_Q},\, H_{v_r}^{(Q)}\right] = \frac{i}{2}\boldsymbol{\theta}\cdot\boldsymbol{\sigma}_{\mathbf{4}\times\mathbf{4}} H_{v_r}^{(Q)}, \tag{2.32}$$

so that

$$\delta P_{v_r}^{(Q)} = -\tfrac{1}{2}\boldsymbol{\theta}\cdot\mathbf{P}_{v_r}^{*(Q)}, \qquad \delta\mathbf{P}_{v_r}^{*(Q)} = \tfrac{1}{2}\boldsymbol{\theta}\times\mathbf{P}_{v_r}^{*(Q)} - \tfrac{1}{2}\boldsymbol{\theta} P_{v_r}^{(Q)}. \tag{2.33}$$

Under finite heavy quark spin transformations,

$$H_v^{(Q)} \to D(R)_Q H_v^{(Q)}, \tag{2.34}$$

where $D(R)_Q$ is the rotation matrix in the spinor representation for the rotation R. Like the Lorentz transformations, it satisfies $\gamma^0 D(R)_Q^\dagger \gamma^0 = D(R)_Q^{-1}$.

It is straightforward to write couplings that are invariant under the heavy quark symmetry using the field $H_v^{(Q)}$ and its transformation rules. We have concentrated on the heavy quark spin symmetry, because that is the new ingredient in the formalism. One can also implement the heavy quark flavor symmetry by using fields $H_v^{(Q_i)}$ for each heavy quark flavor Q_i, and also imposing heavy flavor symmetry

$$H_v^{(Q_i)} \to U_{ij} H_v^{(Q_j)}, \tag{2.35}$$

where U_{ij} is an arbitrary unitary matrix in flavor space.

We have seen how to use a covariant formalism for the pseudoscalar and vector meson multiplet. It is straightforward to derive a similar formalism for baryon states. For example the Λ_Q baryon has light degrees of freedom with spin zero, so the spin of the baryon is the spin of the heavy quark. It is described by a spinor field $\Lambda_v^{(Q)}(x)$ that satisfies the constraint

$$\not{v}\Lambda_v^{(Q)} = \Lambda_v^{(Q)}, \tag{2.36}$$

transforms under the Lorentz group as[†]

$$\Lambda_v^{(Q)}(x) \to D(\Lambda)\,\Lambda_{\Lambda^{-1}v}^{(Q)}(\Lambda^{-1}x), \tag{2.37}$$

and transforms under heavy quark spin transformations as

$$\Lambda_v^{(Q)} \to D(R)_Q \Lambda_v^{(Q)}. \tag{2.38}$$

The analog of the polarization vector for spin-$1/2$ Λ_Q states with velocity v and spin s is the spinor $u(v,s)$. These spinors will be normalized so that

$$\bar{u}(v,s)\gamma^\mu u(v,s) = 2v^\mu. \tag{2.39}$$

[†] We hope the reader is not confused by the use of Λ for both the Lorentz transformation and the heavy baryon field.

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Then

$$\bar{u}(v,s)\gamma^{\mu}\gamma_5 u(v,s) = 2s^{\mu}, \tag{2.40}$$

where s^{μ} is the spin vector, satisfying $v \cdot s = 0$ and $s^2 = -1$. The field $\Lambda_v^{(Q)}$ annihilates heavy baryon states with amplitude $u(v,s)$.

2.6 The effective Lagrangian

The QCD Lagrangian does not have manifest heavy quark spin-flavor symmetry as $m_Q \to \infty$. It is convenient to use an effective field theory for QCD in which heavy quark symmetry is manifest in the $m_Q \to \infty$ limit. This effective field theory is known as heavy quark effective theory (HQET), and it describes the dynamics of hadrons containing a single heavy quark. It is a valid description of the physics at momenta much smaller than the mass of the heavy quark m_Q. The effective field theory is constructed so that only inverse powers of m_Q appear in the effective Lagrangian, in contrast to the QCD Lagrangian in Eq. (1.82), which has positive powers of m_Q.

Consider a single heavy quark with velocity v interacting with external fields, where the velocity of an on-shell quark is defined by $p = m_Q v$. The momentum of an off-shell quark can be written as $p = m_Q v + k$, where the residual momentum k determines the amount by which the quark is off shell because of its interactions. For heavy quarks in a hadron, k is of the order of $\Lambda_{\rm QCD}$. The usual Dirac quark propagator simplifies to

$$i\,\frac{\not{p} + m_Q}{p^2 - m_Q^2 + i\varepsilon} = i\,\frac{m_Q\not{v} + m_Q + \not{k}}{2m_Q v\cdot k + k^2 + i\varepsilon} \to i\,\frac{1+\not{v}}{2v\cdot k + i\varepsilon} \tag{2.41}$$

in the heavy quark limit. The propagator contains a velocity-dependent projection operator

$$\frac{1+\not{v}}{2}. \tag{2.42}$$

In the rest frame of the heavy quark this projection operator becomes $(1+\gamma^0)/2$, which projects onto the particle components of the four-component Dirac spinor.

It is convenient to formulate the effective Lagrangian directly in terms of velocity-dependent fields $Q_v(x)$, which are related to the original quark fields $Q(x)$ at tree level. One can write the original quark field $Q(x)$ as

$$Q\,(x) = e^{-im_Q v\cdot x}[Q_v(x) + \mathfrak{Q}_v(x)]\,, \tag{2.43}$$

where

$$Q_v(x) = e^{im_Q v\cdot x}\,\frac{1+\not{v}}{2}\,Q(x)\,, \qquad \mathfrak{Q}_v(x) = e^{im_Q v\cdot x}\,\frac{1-\not{v}}{2}\,Q(x)\,. \tag{2.44}$$

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The exponential prefactor subtracts $m_Q v^\mu$ from the heavy quark momentum. The Q_v field produces effects at leading order, whereas the effects of $\mathfrak{Q}_v$ are suppressed by powers of $1/m_Q$. These $1/m_Q$ corrections are discussed in Chapter 4. Neglecting $\mathfrak{Q}_v$ and substituting Eq. (2.43) into the part of the QCD Lagrangian density involving the heavy quark field, $\bar{Q}(i\not{D}-m_Q)Q$ gives $\bar{Q}_v i\not{D} Q_v$. Inserting $(1+\not{v})/2$ on either side of $\not{D}$ yields

$$\mathcal{L} = \bar{Q}_v\,(iv\cdot D)\,Q_v, \tag{2.45}$$

which is an m_Q-independent expression. The Q_v propagator that follows from Eq. (2.45) is

$$\left(\frac{1+\not{v}}{2}\right)\frac{i}{(v\cdot k+i\varepsilon)}, \tag{2.46}$$

which is the same as was derived previously by taking the $m_Q\to\infty$ limit of the Feynman rules. The projector in Eq. (2.46) arises because Q_v satisfies

$$\left(\frac{1+\not{v}}{2}\right)Q_v = Q_v. \tag{2.47}$$

Beyond tree level, there is no simple connection between the fields Q_v of the effective Lagrangian and Q of the QCD theory. The effective theory is constructed by making sure that on-shell Green's functions in the effective theory are equal to those in QCD to a given order in $1/m_Q$ and $\alpha_s(m_Q)$. At tree level, we have seen that the quark propagator in the effective theory matches that in the full theory up to terms of the order of $1/m_Q$. It remains to show that the gluon interaction vertex is the same in the two theories. Consider a generic gluon interaction, as shown in Fig. 2.4. The interaction vertex in the full theory is $-igT^A\gamma^\mu$, whereas in the effective theory, the vertex is $-igT^Av^\mu$ from the $v\cdot D$ term in Eq. (2.45). The vertex in the full theory is sandwiched between quark propagators. Each heavy quark propagator is proportional to $\left(1+\not{v}\right)/2$, so the factor of γ^μ in the vertex can be replaced by

$$\gamma^\mu \to \frac{1+\not{v}}{2}\gamma^\mu\frac{1+\not{v}}{2} = v^\mu\frac{1+\not{v}}{2} \to v^\mu, \tag{2.48}$$

which gives the same vertex as in the effective theory. Thus the effective Lagrangian in Eq. (2.45) reproduces all the Green's functions in the full theory to leading order in $1/m_Q$ and $\alpha_s(m_Q)$. If there is more than one heavy quark

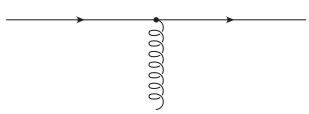

Fig. 2.4. The quark–gluon vertex.

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flavor, the effective Lagrangian at leading order in $1/m_Q$ is

$$\mathcal{L}_{\text{eff}} = \sum_{i=1}^{N_h} \bar{Q}_v^{(i)} \left(iv \cdot D\right) Q_v^{(i)}, \tag{2.49}$$

where N_h is the number of heavy quark flavors and all the heavy quarks have the same four-velocity v. The effective Lagrangian in Eq. (2.49) does not depend on the masses or spins of the heavy quarks, and so has a manifest $U(2N_h)$ spin-flavor symmetry under which the $2N_h$ quark fields transform as the fundamental $2N_h$-dimensional representation. There are only $2N_h$ independent components in the N_h fields $Q_v^{(i)}$, because the constraint in Eq. (2.47) eliminates two of the four components in each $Q_v^{(i)}$ spinor field.

2.7 Normalization of states

The standard relativistic normalization for hadronic states is

$$\langle H(p') | H(p) \rangle = 2E_{\mathbf{p}}\,(2\pi)^3\,\delta^3(\mathbf{p} - \mathbf{p}'), \tag{2.50}$$

where $E_{\mathbf{p}} = \sqrt{|\mathbf{p}|^2 + m_H^2}$. States with the normalization in Eq. (2.50) have mass dimension -1. In HQET, hadron states are labeled by a four-velocity v and a residual momentum k satisfying $v \cdot k = 0$. These states are defined by using the HQET Lagrangian in the $m_Q \to \infty$ limit. They differ from full QCD states by $1/m_Q$ corrections and a normalization factor. The normalization convention in HQET is

$$\langle H(v', k') | H(v, k) \rangle = 2v^0\,\delta_{vv'}\,(2\pi)^3\,\delta^3(\mathbf{k} - \mathbf{k}'). \tag{2.51}$$

Possible spin labels are suppressed in Eqs. (2.50) and (2.51). The split between the four-velocity v and the residual momentum is somewhat arbitrary, and the freedom to redefine v by an amount of order Λ_{QCD}/m_Q while changing k by a corresponding amount of order Λ_{QCD} is called reparameterization invariance. We shall explore the consequences of this freedom in Chapter 4. In matrix elements we shall usually take our initial and final hadron states that contain a single heavy quark to have zero residual momentum and not show explicitly the dependence of the state on the residual momentum; i.e., k will be dropped in the labeling of states, $|H(v)\rangle \equiv |H(v, k=0)\rangle$. The advantage of the normalization in Eq. (2.51) is that it has no dependence on the mass of the heavy quark. A factor of m_H has been removed in comparison with the standard relativistic norm in Eq. (2.50). States normalized by using the HQET convention have mass dimension $-3/2$.

In the remainder of the book, matrix elements in full QCD will be taken between states normalized by using the usual relativistic convention and labeled

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by the momentum p, whereas matrix elements in HQET will be taken between states normalized by using the HQET convention and labeled by their velocity v. The two normalizations differ by a factor $\sqrt{m_H}$,

$$|H(p)\rangle = \sqrt{m_H}\,[|H(v)\rangle + \mathcal{O}(1/m_Q)]. \tag{2.52}$$

Similarly Dirac spinors $u(p, s)$ labeled by momentum are normalized to satisfy

$$\bar{u}(p, s)\gamma^\mu u(p, s) = 2p^\mu, \tag{2.53}$$

and those labeled by velocity to satisfy

$$\bar{u}(v, s)\gamma^\mu u(v, s) = 2v^\mu. \tag{2.54}$$

The spinors $u(p, s)$ and $u(v, s)$ differ by a factor of $\sqrt{m_H}$

$$u(p, s) = \sqrt{m_H}\, u(v, s). \tag{2.55}$$

2.8 Heavy meson decay constants

Heavy meson decay constants are one of the simplest quantities that can be studied with HQET. The pseudoscalar meson decay constants for the $\bar{B}$ and D mesons are defined by[‡]

$$\langle 0|\bar{q}\gamma^\mu\gamma_5 Q(0)\,|P(p)\rangle = -i f_P\, p^\mu, \tag{2.56}$$

where f_P has mass dimension one. Vector meson decay constants for the D^* and $\bar{B}^*$ mesons are defined by

$$\langle 0|\bar{q}\,\gamma^\mu Q(0)\,|P^*(p, \epsilon)\rangle = f_{P^*}\,\epsilon^\mu, \tag{2.57}$$

where ϵ_μ is the polarization vector of the meson. f_{P^*} has mass dimension two.

The vector and axial currents $\bar{q}\gamma^\mu Q$ and $\bar{q}\gamma^\mu\gamma_5 Q$ can be written in terms of HQET fields,

$$\bar{q}\,\Gamma^\mu Q(0) = \bar{q}\,\Gamma^\mu Q_v(0)\,, \tag{2.58}$$

where $\Gamma^\mu = \gamma^\mu$ or $\gamma^\mu\gamma_5$. There are $\alpha_s(m_Q)$ and $1/m_Q$ corrections to this matching condition, which will be discussed in Chapters 3 and 4, respectively.

The matrix elements required in the heavy quark effective theory are

$$\langle 0|\bar{q}\,\Gamma^\mu Q_v(0)\,|H(v)\rangle, \tag{2.59}$$

where $|H(v)\rangle$ denotes either the P or P^* states with zero residual momentum, normalized using Eq. (2.51). For these matrix elements, it is helpful to reexpress the current $\bar{q}\,\Gamma^\mu Q_v$ in terms of the hadron field $H_v^{(Q)}$ of Eq. (2.19). The current

[‡] The pion decay constant f_π defined with the normalization convention in Eq. (2.56) has a value of 131 MeV.

$\bar{q}\,\Gamma^\mu Q_v$ is a Lorentz four vector that transforms as

$$\bar{q}\,\Gamma^\mu Q_v \to \bar{q}\,\Gamma^\mu D(R)_Q Q_v \tag{2.60}$$

under heavy quark spin transformations, where $D(R)_Q$ is the rotation matrix for a heavy quark field. The representation of the current in terms of $H_v^{(Q)}$ should transform in the same manner as Eq. (2.60) under heavy quark spin transformations. This can be done by using a standard trick: (i) Pretend that Γ^μ transforms as $\Gamma^\mu \to \Gamma^\mu D(R)_Q^{-1}$ so that the current is an invariant. (ii) Write down operators that are invariant when $Q_v \to D(R)_Q Q_v$, $\Gamma^\mu \to \Gamma^\mu D(R)_Q^{-1}$, and $H_v^{(Q)} \to D(R)_Q H_v^{(Q)}$. (iii) Set Γ^μ to its fixed value γ^μ or $\gamma^\mu\gamma_5$ to obtain the operator with the correct transformation properties.

The current must have a single $H_v^{(Q)}$ field, since the matrix element in Eq. (2.59) contains a single initial-state heavy meson. The field $H_v^{(Q)}$ and Γ^μ can only occur as the product $\Gamma^\mu H_v^{(Q)}$ for the current to be invariant under heavy quark spin symmetry. For Lorentz covariance, the current must have the form

$$\mathrm{Tr}\; X\Gamma^\mu H_v^{(Q)}, \tag{2.61}$$

where X is a Lorentz bispinor. The only parameter that X can depend on is v, so X must have the form $a_0(v^2) + a_1(v^2)\not{v}$, by Lorentz covariance and parity. All dependence on spin has already been included in the indices of the H field, so X can have no dependence on the polarization of the P^* meson. Since $H_v^{(Q)}\not{v} = -H_v^{(Q)}$ and $v^2 = 1$, one can write

$$\bar{q}\Gamma^\mu Q_v = \frac{a}{2}\mathrm{Tr}\;\Gamma^\mu H_v^{(Q)}, \tag{2.62}$$

where $a = [a_0(1) - a_1(1)]$ is an unknown normalization constant that is independent of the mass of the heavy quark Q. Evaluating the trace explicitly gives

$$a \times \begin{cases} -i\,v^\mu P_v^{(Q)} & \text{if } \Gamma^\mu = \gamma^\mu\gamma_5, \\ P_v^{*(Q)\mu} & \text{if } \Gamma^\mu = \gamma^\mu, \end{cases} \tag{2.63}$$

where $P_v^{(Q)}$ and $P_{v\mu}^{*(Q)}$ are the pseudoscalar and vector fields that destroy the corresponding hadrons. The resulting matrix elements are

$$\begin{aligned} \langle 0|\bar{q}\gamma^\mu\gamma_5 Q_v|P(v)\rangle &= -i a v^\mu, \\ \langle 0|\bar{q}\gamma^\mu Q_v|P^*(v)\rangle &= a\epsilon^\mu. \end{aligned} \tag{2.64}$$

Comparing with the definitions of the meson decay constants Eqs. (2.56), (2.57), and using $p^\mu = m_{P^{(*)}} v^\mu$ gives the relations

$$f_P = \frac{a}{\sqrt{m_P}}, \qquad f_{P^*} = a\sqrt{m_{P^*}}. \tag{2.65}$$

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Table 2.3. *Heavy meson decay constants from a lattice Monte Carlo simulation*[a]

Decay Constant	Value in MeV
f_D	197 ± 2
f_{D_s}	224 ± 2
f_B	173 ± 4
f_{B_s}	199 ± 3

[a] From the JLQCD Collaboration [S. Aoki et al., Phys. Rev. Lett. 80 (1998), 5711]. Only the statistical errors are quoted.

The factors of $\sqrt{m_P}$ and $\sqrt{m_{P^*}}$ are due to the difference between the normalizations of states in Eqs. (2.50) and (2.51). The P and P^* masses are equal in the heavy quark limit, so one can write the equivalent relations

$$f_P = \frac{a}{\sqrt{m_P}}, \qquad f_{P^*} = m_P f_P, \tag{2.66}$$

which imply that $f_P \propto m_P^{-1/2}$ and $f_{P^*} \propto m_P^{1/2}$. For the D and B system, one finds

$$\frac{f_B}{f_D} = \sqrt{\frac{m_D}{m_B}}, \quad f_{D^*} = m_D f_D, \quad f_{B^*} = m_B f_B. \tag{2.67}$$

The decay constants for the pseudoscalar mesons can be measured by means of the weak leptonic decays $D \to \bar{\ell}\nu_\ell$ and $\bar{B} \to \ell\bar{\nu}_\ell$. The partial width is

$$\Gamma = \frac{G_F^2 |V_{Qq}|^2}{8\pi} f_P^2 m_\ell^2 m_P \left(1 - \frac{m_\ell^2}{m_P^2}\right)^2. \tag{2.68}$$

The only heavy meson decay constant that has been measured is f_{D_s}, from the decays $D_s^+ \to \bar{\mu}\nu_\mu$ and $D_s^+ \to \bar{\tau}\nu_\tau$. However, at the present time, the reported values vary over a large range of $\sim$200–300 MeV. Values of the heavy meson decay constants determined from a lattice Monte Carlo simulation of QCD are shown in Table 2.3. Only statistical errors are quoted. Note that this simulation suggests that there is a substantial correction to the heavy quark symmetry prediction $f_B/f_D = \sqrt{m_D/m_B} \simeq 0.6$.

2.9 $\bar{B} \to D^{(*)}$ form factors

The semileptonic decays of a $\bar{B}$ meson to D and D^* mesons allow one to determine the weak mixing angle V_{cb}. The semileptonic $\bar{B}$ meson decay amplitude is

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determined by the matrix elements of the weak Hamiltonian:

$$H_W = \frac{4G_F}{\sqrt{2}} V_{cb}[\bar{c}\gamma_\mu P_L b][\bar{e}\gamma^\mu P_L \nu_e]. \tag{2.69}$$

Neglecting higher-order electroweak corrections, the matrix element factors into the product of leptonic and hadronic matrix elements. The hadronic part is the matrix element of the vector or axial vector currents $V^\mu = \bar{c}\gamma^\mu b$ and $A^\mu = \bar{c}\gamma^\mu\gamma_5 b$ between $\bar{B}$ and $D^{(*)}$ states.

It is convenient to write the most general possible matrix element in terms of a few Lorentz invariant amplitudes called form factors. The most general vector current matrix element for $\bar{B} \to D$ must transform as a Lorentz four vector. The only four vectors in the problem are the momenta p and p' of the initial and final mesons, so the matrix element must have the form $ap^\mu + bp'^\mu$. The form factors a and b are Lorentz invariant functions that can only depend on the invariants in the problem, p^2, p'^2 and $p \cdot p'$. Two of the variables are fixed, $p^2 = m_B^2$ and $p'^2 = m_D^2$, and it is conventional to choose $q^2 = (p - p')^2$ as the only independent variable. A similar analysis can be carried out for the other matrix elements. The amplitudes involving the D^* are linear in its polarization vector ϵ and can be simplified by noting that the polarization vector satisfies the constraint $p' \cdot \epsilon = 0$. The conventional choice of form factors allowed by parity and time-reversal invariance is

$$\begin{aligned}
\langle D(p')|V^\mu|\bar{B}(p)\rangle &= f_+(q^2)(p+p')^\mu + f_-(q^2)(p-p')^\mu, \\
\langle D^*(p',\epsilon)|V^\mu|\bar{B}(p)\rangle &= g(q^2)\epsilon^{\mu\nu\alpha\tau}\epsilon^*_\nu(p+p')_\alpha(p-p')_\tau, \\
\langle D^*(p',\epsilon)|A^\mu|\bar{B}(p)\rangle &= -if(q^2)\epsilon^{*\mu} \\
&\quad -i\epsilon^*\cdot p[a_+(q^2)(p+p')^\mu + a_-(q^2)(p-p')^\mu],
\end{aligned} \tag{2.70}$$

where $q = p - p'$, all the form factors are real, and the states have the usual relativistic normalization.

Under parity and time reversal,

$$\begin{aligned}
P|D(p)\rangle &= -|D(p_P)\rangle, & T|D(p)\rangle &= -|D(p_T)\rangle, \\
P|D^*(p,\epsilon)\rangle &= |D^*(p_P,\epsilon_P)\rangle, & T|D^*(p,\epsilon)\rangle &= |D^*(p_T,\epsilon_T)\rangle,
\end{aligned} \tag{2.71}$$

which are the usual transformations for pseudoscalar and vector particles. Here $p = (p^0, \mathbf{p})$, $\epsilon = (\epsilon^0, \boldsymbol{\epsilon})$, and $p_P = p_T = (p^0, -\mathbf{p})$, $\epsilon_P = \epsilon_T = (\epsilon^0, -\boldsymbol{\epsilon})$. Analogous equations hold for the $\bar{B}$ and $\bar{B}^*$. Parity and time-reversal invariance of the strong interactions implies that the matrix elements of currents between two states $|\psi\rangle$ and $|\chi\rangle$ transform as

$$\begin{aligned}
\langle\psi|J^0|\chi\rangle &= \eta_P\langle\psi_P|J^0|\chi_P\rangle, & \langle\psi|J^0|\chi\rangle^* &= \eta_T\langle\psi_T|J^0|\chi_T\rangle, \\
\langle\psi|J^i|\chi\rangle &= -\eta_P\langle\psi_P|J^i|\chi_P\rangle, & \langle\psi|J^i|\chi\rangle^* &= -\eta_T\langle\psi_P|J^i|\chi_P\rangle,
\end{aligned} \tag{2.72}$$

where $\eta_P = 1,\ \eta_T = 1$ if J is the vector current, $\eta_P = -1,\ \eta_T = 1$ if J is the axial current, and $|\chi_P\rangle \equiv P|\chi\rangle$, $|\chi_T\rangle \equiv T|\chi\rangle$, and so on. One can now show that Eq. (2.70) is the most general form factor decomposition. Consider, for example, $\langle D^*(p',\epsilon)|V^\mu|\bar{B}(p)\rangle$. Parity invariance requires that

$$\langle D^*(p',\epsilon)|V^0|\bar{B}(p)\rangle = -\langle D^*(p'_P,\epsilon_P)|V^0|\bar{B}(p_P)\rangle. \tag{2.73}$$

The only possible tensor combination that changes sign under parity is $\epsilon^{0\nu\alpha\tau}\epsilon^*_\nu p_\alpha\ p'_\tau$, which is proportional to the right-hand side in Eq. (2.70). Time-reversal invariance requires

$$\langle D^*(p',\epsilon)|V^0|\bar{B}(p)\rangle^* = -\langle D^*(p'_T,\epsilon_T)|V^0|\bar{B}(p_T)\rangle, \tag{2.74}$$

which implies that $g(q^2)$ is real. One can similarly work through the other two cases. The factors of i in Eq. (2.70) depend on the phase convention for the meson states. We have chosen to define the pseudoscalar state to be odd under time reversal. Another choice used is i times this, which corresponds to a state which is even under time reversal. This introduces a factor of i in the last two matrix elements in Eq. (2.70).

It is straightforward to express the differential decay rates $\mathrm{d}\Gamma(\bar{B} \to D^{(*)}e\bar{\nu}_e)/\mathrm{d}q^2$ in terms of the form factors $f_\pm$, f, g, and $a_\pm$. To a very good approximation, the electron mass can be neglected, and consequently a_- and f_- do not contribute to the differential decay rate. For $\bar{B} \to De\bar{\nu}_e$ the invariant decay matrix element is

$$\mathcal{M}(\bar{B} \to De\bar{\nu}_e) = \sqrt{2}G_F V_{cb}\ f_+\,(p+p')^\mu\ \bar{u}(p_e)\gamma_\mu P_L v\big(p_{\nu_e}\big). \tag{2.75}$$

Squaring and summing over electron spins yields,

$$\begin{aligned}|\mathcal{M}|^2 &= \sum_{\text{spins}} |\mathcal{M}(\bar{B} \to De\bar{\nu}_e)|^2 \\ &= 2G_F^2|V_{cb}|^2|f_+|^2(p+p')^{\mu_1}(p+p')^{\mu_2}\,\mathrm{Tr}\left[\not{p}_e\gamma_{\mu_1}\not{p}_{\nu_e}\gamma_{\mu_2}P_L\right].\end{aligned} \tag{2.76}$$

The differential decay rate is

$$\begin{aligned}\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}(\bar{B} \to De\bar{\nu}_e) &= \frac{1}{2m_B}\int \frac{d^3p'}{(2\pi)^3 2p'^0}\int \frac{d^3p_e}{(2\pi)^3 2p_e^0} \\ &\times \int \frac{d^3p_{\nu_e}}{(2\pi)^3 2p^0_{\nu_e}}|\mathcal{M}|^2(2\pi)^4\delta^4\big(q - p_e - p_{\nu_e}\big)\delta[q^2-(p-p')^2],\end{aligned} \tag{2.77}$$

where q^2 is the hadronic momentum transfer squared, or equivalently, the invariant mass squared of the lepton pair. The integration measure is symmetric with respect to electron and neutrino momenta, so the part of the trace in Eq. (2.76) involving γ_5 does not contribute. It would contribute to the electron spectrum

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$\mathrm{d}\Gamma(\bar{B} \to De\bar{\nu}_e)/\mathrm{d}E_e$. The integration over electron and neutrino momenta gives

$$\int \frac{d^3 p_e}{(2\pi)^3 2p_e^0} \int \frac{d^3 p_{\nu_e}}{(2\pi)^3 2p_{\nu_e}^0} \mathrm{Tr}\big[\not{p}_e \gamma_{\mu_1} \not{p}_{\nu_e} \gamma_{\mu_2}\big](2\pi)^4 \delta^4\big[q - \big(p_e + p_{\nu_e}\big)\big]$$
$$= \frac{1}{6\pi}\big(q_{\mu_1} q_{\mu_2} - g_{\mu_1\mu_2} q^2\big). \tag{2.78}$$

Finally, using

$$(p + p')^{\mu_1}(p + p')^{\mu_2}\big(q_{\mu_1} q_{\mu_2} - g_{\mu_1\mu_2} q^2\big)$$
$$= \big(q^2 - m_B^2 - m_D^2\big)^2 - 4m_B^2 m_D^2, \tag{2.79}$$

and the two-body phase space formula

$$\int \frac{d^3 p'}{(2\pi)^3 2p'^0} \delta[q^2 - (p - p')^2] = \frac{1}{16\pi^2 m_B^2} \sqrt{\big(q^2 - m_B^2 - m_D^2\big)^2 - 4m_B^2 m_D^2}, \tag{2.80}$$

the differential decay rate in Eq. (2.77) becomes

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}(\bar{B} \to De\bar{\nu}_e) = \frac{G_F^2 |V_{cb}|^2 |f_+|^2}{192\pi^3 m_B^3}\big[\big(q^2 - m_B^2 - m_D^2\big)^2 - 4m_B^2 m_D^2\big]^{3/2}. \tag{2.81}$$

A similar but more complicated expression holds for $\mathrm{d}\Gamma(\bar{B} \to D^* e\bar{\nu}_e)/\mathrm{d}q^2$.

It is convenient, for comparing with the predictions of HQET, not to write the $\bar{B} \to D^{(*)}$ matrix elements of the vector and axial vector current as in Eq. (2.70), but rather to introduce new form factors that are linear combinations of $f_\pm$, f, g, and $a_\pm$. The four velocities of the $\bar{B}$ and $D^{(*)}$ mesons are $v^\mu = p^\mu/m_B$ and $v'^\mu = p'^\mu/m_{D^{(*)}}$, and the dot product of these four velocities, $w = v \cdot v'$, is related to q^2 by

$$w = v \cdot v' = \big[m_B^2 + m_{D^{(*)}}^2 - q^2\big]/[2m_B m_{D^{(*)}}]. \tag{2.82}$$

The allowed kinematic range for w is

$$0 \le w - 1 \le [m_B - m_{D^{(*)}}]^2/[2m_B m_{D^{(*)}}]. \tag{2.83}$$

The zero-recoil point, at which $D^{(*)}$ is at rest in the $\bar{B}$ rest frame, is $w = 1$. The new form factors $h_\pm$, h_V, and h_{A_j} are expressed as functions of w instead of q^2 and are defined by

$$\begin{aligned}
\frac{\langle D(p')|V^\mu|\bar{B}(p)\rangle}{\sqrt{m_B m_D}} &= h_+(w)\,(v + v')^\mu + h_-(w)\,(v - v')^\mu, \\
\frac{\langle D^*(p', \epsilon)|V^\mu|\bar{B}(p)\rangle}{\sqrt{m_B m_{D^*}}} &= h_V(w)\epsilon^{\mu\nu\alpha\beta}\epsilon_\nu^* v'_\alpha v_\beta, \\
\frac{\langle D^*(p', \epsilon)|A^\mu|\bar{B}(p)\rangle}{\sqrt{m_B m_{D^*}}} &= -ih_{A_1}(w)(w + 1)\epsilon^{*\mu} + ih_{A_2}(w)(\epsilon^* \cdot v)v^\mu \\
&\quad + ih_{A_3}(w)\,(\epsilon^* \cdot v)v'^\mu.
\end{aligned} \tag{2.84}$$

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The differential decay rates $\mathrm{d}\Gamma(\bar{B} \to D^{(*)} e \bar{\nu}_e)/\mathrm{d}w$ in terms of these form factors are

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}w}(\bar{B} \to D e \bar{\nu}_e) = \frac{G_F^2 |V_{cb}|^2 m_B^5}{48\pi^3}(w^2-1)^{3/2} r^3 (1+r)^2 \mathcal{F}_D(w)^2,$$

$$\begin{aligned}\frac{\mathrm{d}\Gamma}{\mathrm{d}w}(\bar{B} \to D^* e \bar{\nu}_e) &= \frac{G_F^2 |V_{cb}|^2 m_B^5}{48\pi^3}(w^2-1)^{1/2}(w+1)^2 r^{*3}(1-r^*)^2 \\ &\quad \times \left[1 + \frac{4w}{w+1}\frac{1-2wr^*+r^{*2}}{(1-r^*)^2}\right]\mathcal{F}_{D^*}(w)^2,\end{aligned} \tag{2.85}$$

where

$$r = \frac{m_D}{m_B}, \qquad r^* = \frac{m_{D^*}}{m_B}, \tag{2.86}$$

and

$$\begin{aligned}\mathcal{F}_D(w)^2 &= \left[h_+ + \left(\frac{1-r}{1+r}\right)h_-\right]^2, \\ \mathcal{F}_{D^*}(w)^2 &= \left\{2(1-2wr^*+r^{*2})\left[h_{A_1}^2 + \left(\frac{w-1}{w+1}\right)h_V^2\right]\right. \\ &\quad \left. + \left[(1-r^*)h_{A_1} + (w-1)\big(h_{A_1} - h_{A_3} - r^* h_{A_2}\big)\right]^2\right\} \\ &\quad \times \left\{(1-r^*)^2 + \frac{4w}{w+1}(1-2wr^*+r^{*2})\right\}^{-1}.\end{aligned} \tag{2.87}$$

The spin-flavor symmetry of heavy quark effective theory can be used to derive relations between the form factors $h_\pm$, h_V, and h_{A_j}. A transition to the heavy quark effective theory is possible provided the typical momentum transfer to the light degrees of freedom is small compared to the heavy quark masses. In $\bar{B} \to D^{(*)} e \bar{\nu}_e$ semileptonic decay, q^2 is not small compared with $m_{c,b}^2$. However, this variable does not determine the typical momentum transfer to the light degrees of freedom. A rough measure of that is the momentum transfer that must be given to the light degrees of freedom so that they recoil with the $D^{(*)}$. The light degrees of freedom in the initial and final hadrons have momentum of order $\Lambda_{\mathrm{QCD}} v$ and $\Lambda_{\mathrm{QCD}} v'$, respectively, since their velocity is fixed to be the same as the heavy quark velocity. The momentum transfer for the light system is then $q_{\mathrm{light}}^2 \sim (\Lambda_{\mathrm{QCD}} v - \Lambda_{\mathrm{QCD}} v')^2 = 2\Lambda_{\mathrm{QCD}}^2(1-w)$. Heavy quark symmetry should hold, provided

$$2\Lambda_{\mathrm{QCD}}^2\,(w-1) \ll m_{b,c}^2. \tag{2.88}$$

The heavy meson form factors are expected to vary on the scale $q_{\mathrm{light}}^2 \sim \Lambda_{\mathrm{QCD}}^2$, i.e., on the scale $w \sim 1$.

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The six form factors can be computed in terms of a single function using heavy quark symmetry. The QCD matrix elements required are of the form $\langle H^{(c)}(p')|\bar{c}\,\Gamma b|H^{(b)}(p)\rangle$, where $\Gamma = \gamma^\mu$, $\gamma^\mu\gamma_5$ and $H^{(Q)}$ is either $P^{(Q)}$ or $P^{*(Q)}$. At leading order in $1/m_{c,b}$ and $\alpha_s(m_{c,b})$, the current $\bar{c}\,\Gamma b$ can be replaced by the current $\bar{c}_{v'}\Gamma b_v$ involving heavy quark fields and the heavy mesons states $|H^{(Q)}(p^{(\prime)})\rangle$ by the corresponding ones in HQET $|H^{(Q)}(v^{(\prime)})\rangle$. One can then use a trick similar to that used for the meson decay constants: the current is invariant under spin transformations on the $c_{v'}$ and b_v quark fields, provided that Γ transforms as $D(R)_c\Gamma D(R)_b^{-1}$ where $D(R)_c$ and $D(R)_b$ are the heavy quark spin rotation matrices for c and b quarks, respectively. For the required matrix elements one represents the current by operators that contain one factor each of $\bar{H}_{v'}^{(c)}$ and $H_v^{(b)}$, so that a meson containing a b quark is converted to one containing a c quark. Invariance under the b and c quark spin symmetries requires that the operators should be of the form $\bar{H}_{v'}^{(c)}\Gamma H_v^{(b)}$, so that the factors of $D(R)_{b,c}$ cancel between the Γ matrix and the H fields. Lorentz covariance then requires that

$$\bar{c}_{v'}\Gamma b_v = \mathrm{Tr}\, X \bar{H}_{v'}^{(c)}\Gamma H_v^{(b)}, \tag{2.89}$$

where X is the most general possible bispinor that one can construct using the available variables, v and v'. The most general form for X with the correct parity and time-reversal properties is

$$X = X_0 + X_1 \not{v} + X_2 \not{v}' + X_3 \not{v}\not{v}', \tag{2.90}$$

where the coefficients are functions of $w = v \cdot v'$. Other allowed terms can all be written as linear combinations of the X_i. For example, $\not{v}'\not{v} = 2w - \not{v}\not{v}'$, and so on. The relations $\not{v} H_v^{(b)} = H_v^{(b)}$ and $\not{v}' \bar{H}_{v'}^{(c)} = -\bar{H}_{v'}^{(c)}$ imply that all the terms in Eq. (2.90) are proportional to the first, so one can write

$$\bar{c}_{v'}\Gamma\, b_v = -\xi(w)\mathrm{Tr}\, \bar{H}_{v'}^{(c)}\,\Gamma H_v^{(b)}, \tag{2.91}$$

where the coefficient is conventionally written as $-\xi(w)$. Evaluating the trace in Eq. (2.91) gives the required HQET matrix elements

$$\begin{aligned}
\langle D(v')|\bar{c}_{v'}\,\gamma_\mu\, b_v|\bar{B}(v)\rangle &= \xi(w)\,[v_\mu + v'_\mu], \\
\langle D^*(v',\epsilon)|\bar{c}_{v'}\gamma_\mu\gamma_5 b_v|\bar{B}(v)\rangle &= -i\xi(w)\,[(1+w)\epsilon^*_\mu - (\epsilon^*\cdot v)v'_\mu], \\
\langle D^*(v',\epsilon)|\bar{c}_{v'}\gamma_\mu b_v|\bar{B}(v)\rangle &= \xi(w)\,\epsilon_{\mu\nu\alpha\beta}\epsilon^{*\nu}v'^\alpha v^\beta.
\end{aligned} \tag{2.92}$$

Equations (2.92) are the implications of heavy quark spin symmetry for the $\bar{B} \to D^{(*)}$ matrix elements of the axial vector and vector currents. The function

$\xi(w)$ is independent of the charm and bottom quark masses. Heavy quark flavor symmetry implies the normalization condition

$$\xi(1) = 1. \tag{2.93}$$

To derive this result, consider the forward matrix element of the vector current $\bar{b}\gamma^\mu b$ between $\bar{B}$ meson states. To leading order in $1/m_b$, the operator $\bar{b}\gamma^\mu b$ can be replaced by $\bar{b}_v\gamma_\mu b_v$. The forward matrix element can then be obtained from Eq. (2.92) by setting $v' = v$, and letting $c \to b$, $D \to \bar{B}$,

$$\frac{\langle \bar{B}(p)|\bar{b}\gamma_\mu b|\bar{B}(p)\rangle}{m_B} = \langle \bar{B}(v)|\bar{b}_v\gamma_\mu b_v|\bar{B}(v)\rangle = 2\,\xi(w=1)\,v_\mu. \tag{2.94}$$

Note that $\xi(w)$ is independent of the quark masses, and so has the same value in Eqs. (2.92) and (2.94). Equivalently, heavy quark flavor symmetry allows one to replace D by $\bar{B}$ in Eq. (2.92). The left-hand side of Eq. (2.94) with $\mu = 0$ is the matrix element of b-quark number between $\bar{B}$ mesons, and so has the value $2v_0$. This implies that $\xi(1) = 1$.

Functions of $w = v \cdot v'$ like ξ occur often in the analysis of matrix elements and are called Isgur-Wise functions. Eq. (2.92) predicts relations between the form factors in Eqs. (2.84):

$$\begin{gathered} h_+(w) = h_V(w) = h_{A_1}(w) = h_{A_3}(w) = \xi(w), \\ h_-(w) = h_{A_2}(w) = 0. \end{gathered} \tag{2.95}$$

This equation implies that

$$\mathcal{F}_D(w) = \mathcal{F}_{D^*}(w) = \xi(w). \tag{2.96}$$

There is experimental support for the utility of the $m_{c,b} \to \infty$ limit for describing $\bar{B} \to D^{(*)}e\bar{\nu}_e$ decays. Figure 2.5 shows a plot of the ratio $\mathcal{F}_{D^*}(w)/\mathcal{F}_D(w)$ as a

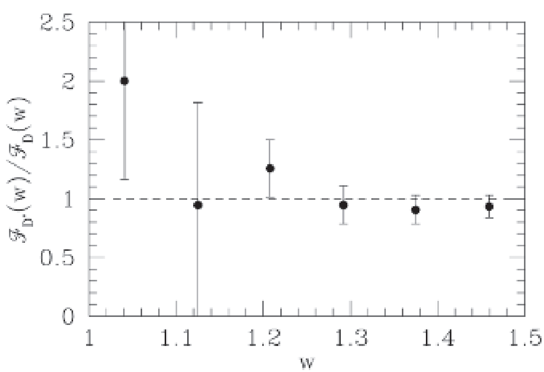

Fig. 2.5. The measured ratio $\mathcal{F}_{D^*}(w)/\mathcal{F}_D(w)$ as a function of w. The data are from the ALEPH Collaboration [D. Buskulic et al., Phys. Lett. B395 (1997) 373].

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function of w using data from the ALEPH collaboration. It shows that $\mathcal{F}_{D^*}(w)$ is indeed near $\mathcal{F}_D(w)$. Note that the experimental errors become large as w approaches unity. This is partly because the differential rates $d\Gamma/dw$ vanish at $w = 1$. In addition to comparing the D and D^* decay rates, there is experimental information on the individual form factors in $\bar{B} \to D^* e\bar{\nu}_e$ decay. It is convenient to define two ratios of these form factors:

$$R_1 = \frac{h_V}{h_{A_1}}, \quad R_2 = \frac{h_{A_3} + r h_{A_2}}{h_{A_1}}. \tag{2.97}$$

In the $m_{c,b} \to \infty$ limit, heavy quark spin symmetry implies that $R_1 = R_2 = 1$. Assuming the form factors h_j have the same shape in w, the CLEO collaboration has obtained the experimental values [J. E. Duboscq et al., Phys. Rev. Lett. 76 (1996) 3898]

$$R_1 = 1.18 \pm 0.3, \quad R_2 = 0.71 \pm 0.2. \tag{2.98}$$

There is a simple physical reason why a single Isgur-Wise function is needed for the matrix elements in Eq. (2.92). In the $m_{c,b} \to \infty$ limit, the spin of the light degrees of freedom is a good quantum number. Since $\bar{c}_{v'}\Gamma b_v$ does not act on the light degrees of freedom, their helicity, h_ℓ, is conserved in the transitions it mediates. For $\bar{B} \to D^{(*)}$ matrix elements, there are two helicity amplitudes corresponding to $h_\ell = 1/2$ and $h_\ell = -1/2$. However, they must be equal by parity invariance and therefore there is only one Isgur-Wise function. There are cases when more than one Isgur-Wise function occurs. For example, in $\Omega_b \to \Omega_c^{(*)} e\bar{\nu}_e$ decay, the initial and final hadrons have $s_\ell = 1$. Thus there are two independent helicity amplitudes $h_\ell = 0$ and $h_\ell = \pm 1$, and consequently, two Isgur-Wise functions occur (see Problem 10).

2.10 $\Lambda_c \to \Lambda$ form factors

Another interesting application of heavy quark symmetry is to the weak decay $\Lambda_c \to \Lambda \bar{e}\nu_e$. This decay is an example of a heavy $\to$ light transition, in which a heavy quark decays to a light quark. The most general weak decay form factors can be written in the form

$$\begin{aligned}
\langle \Lambda(p', s') | \bar{s}\gamma^\mu c | \Lambda_c(p, s) \rangle &= \bar{u}(p', s')[f_1\gamma^\mu + i f_2 \sigma^{\mu\nu} q_\nu + f_3 q^\mu] u(p, s)\,, \\
\langle \Lambda(p', s') | \bar{s}\gamma^\mu \gamma_5 c | \Lambda_c(p, s) \rangle &= \bar{u}(p', s')[g_1\gamma^\mu + i g_2 \sigma^{\mu\nu} q_\nu + g_3 q^\mu] \gamma_5 u(p, s)\,,
\end{aligned} \tag{2.99}$$

where $q = p - p'$ and $\sigma_{\mu\nu} = i[\gamma_\mu, \gamma_\nu]/2$. The form factors f_i and g_i are functions of q^2. Heavy quark spin symmetry on the c-quark constrains the general form

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factor decomposition in Eq. (2.99). Making the transition to HQET, one can write the left-hand side of Eq. (2.99) as

$$\langle \Lambda(p', s')|\bar{s}\,\Gamma c_v|\Lambda_c(v, s)\rangle, \tag{2.100}$$

where $\bar{s}\Gamma c \to \bar{s}\Gamma c_v$ at leading order in $1/m_c$. The matrix element in Eq. (2.100) has the same form factor expansion as Eq. (2.99) with $u(p, s) \to u(v, s)$. The $\sqrt{m_{\Lambda_c}}$ difference between Eqs. (2.99) and (2.100) in the normalization of states is compensated by the same factor in the normalization of spinors. The most general form for the matrix element in Eq. (2.100) consistent with spin symmetry on the c quark is

$$\langle \Lambda(p', s')|\bar{s}\Gamma c_v|\Lambda_c(v, s)\rangle = \bar{u}(p', s')X\Gamma u(v, s)\,, \tag{2.101}$$

where X is the most general bispinor that can be constructed out of p' and v. Note that s and s' cannot be used, because the fermion spin is encoded in the matrix indices of the spinors. The decomposition of X is

$$X = F_1 + F_2 \not{v}, \tag{2.102}$$

where F_i are functions of $v \cdot p'$, and we have used the constraints

$$\not{v} u(v, s) = u(v, s)\,, \qquad \not{p}' u(p', s') = m_\Lambda u(p', s') \tag{2.103}$$

to reduce the number of independent terms. Substituting Eq. (2.102) into Eq. (2.101) and comparing with Eq. (2.99) gives the relations

$$\begin{aligned} f_1 = g_1 &= F_1 + \frac{m_\Lambda}{m_{\Lambda_c}} F_2, \\ f_2 = f_3 = g_2 = g_3 &= \frac{1}{m_{\Lambda_c}} F_2, \end{aligned} \tag{2.104}$$

so that the six form factors f_i, g_i can be written in terms of two functions $F_{1,2}$. The heavy $\to$ light form factors $F_{1,2}$ are expected to vary on the scale $v \cdot p' \sim \Lambda_{\rm QCD}$.

These relations between form factors have implications for the polarization of the Λ's produced in Λ_c decay. Equation (2.104) implies that in the $m_c \to \infty$ limit, the polarization variable

$$\alpha = -\left.\frac{2 f_1 g_1}{f_1^2 + g_1^2}\right|_{q^2=0} \tag{2.105}$$

is equal to -1. The CLEO Collaboration [G. Crawford et al., Phys. Rev. Lett. 75 (1995) 624] finds that, averaged over all q^2, $\alpha = -0.82 \pm 0.10$ consistent with expectations based on charm quark spin symmetry.

2.11 $\Lambda_b \to \Lambda_c$ form factors

The semileptonic weak decay $\Lambda_b \to \Lambda_c e \bar{\nu}_e$ form factors are even more constrained by heavy quark symmetry than the $\Lambda_c \to \Lambda \bar{e} \nu_e$ form factors discussed above, because one can use heavy quark symmetry on both the initial and final baryons. The most general weak decay form factors for $\Lambda_b \to \Lambda_c$ decay are conventionally written as

$$\begin{aligned} \langle \Lambda_c(p', s') | \bar{c} \gamma^\mu b | \Lambda_b(p, s) \rangle &= \bar{u}(p', s') [f_1 \gamma^\mu + f_2 v^\mu + f_3 v'^\mu] u(p, s), \\ \langle \Lambda_c(p', s') | \bar{c} \gamma^\mu \gamma_5 b | \Lambda_b(p, s) \rangle &= \bar{u}(p', s') [g_1 \gamma^\mu + g_2 v^\mu + g_3 v'^\mu] \gamma_5 u(p, s), \end{aligned} \tag{2.106}$$

where f_i and g_i are functions of w. We have taken the general decomposition from Eq. (2.99) and rewritten q^μ and $\sigma^{\mu\nu} q_\nu$ in terms of γ^μ, v^μ and v'^μ. Making the transition to HQET, the matrix element

$$\langle \Lambda_c(v', s') | \bar{c}_{v'} \, \Gamma \, b_v | \Lambda_b(v, s) \rangle = \zeta(w) \bar{u}(v', s') \Gamma u(v, s) \tag{2.107}$$

by heavy quark spin symmetry on the b and c quark fields. Thus we obtain

$$f_1(w) = g_1(w) = \zeta(w), \qquad f_2 = f_3 = g_2 = g_3 = 0. \tag{2.108}$$

The six form factors can be written in terms of the single Isgur-Wise function $\zeta(w)$. As in the meson case

$$\zeta(1) = 1, \tag{2.109}$$

since the form factor of $\bar{b} \gamma^\mu b$ for $\Lambda_b \to \Lambda_b$ transitions at $w = 1$ is b-quark number. The heavy $\to$ heavy relations in Eq. (2.108) are a special case of the heavy $\to$ light relations in Eq. (2.104), with the additional restrictions $F_2 = 0$ and $F_1(v \cdot v' = 1) = 1$.

2.12 Problems

1. In the $m_Q \to \infty$ limit, show that the propagator for a heavy antiquark with momentum $p_{\bar{Q}} = m_Q v + k$ is

$$\frac{i}{v \cdot k + i\varepsilon} \left(\frac{1 - \not{v}}{2} \right),$$

while the heavy antiquark–gluon vertex is

$$i g (T^A)^T v_\mu .$$

2. Compare the theoretical expectation for the ratio $\Gamma(D_1 \to D^* \pi) / \Gamma(D_2^* \to D^* \pi)$ with its experimental value. Discuss your result.

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3. Consider the following heavy-light matrix elements of the vector and axial vector currents

$$\begin{aligned}\langle V(p',\epsilon)|\bar{q}\gamma_\mu\gamma_5 Q|P^{(Q)}(p)\rangle &= -if^{(Q)}\epsilon^*_\mu - i\epsilon^*\cdot p\big[a_+^{(Q)}(p+p')_\mu + a_-^{(Q)}(p-p')_\mu\big],\\ \langle V(p',\epsilon)|\bar{q}\gamma_\mu Q|P^{(Q)}(p)\rangle &= g^{(Q)}\epsilon_{\mu\nu\lambda\sigma}\epsilon^{*\nu}(p+p')^\lambda(p-p')^\sigma,\end{aligned}$$

where $p = m_{P^{(Q)}}v$. The form factors $f^{(Q)}$, $a_\pm^{(Q)}$ and $g^{(Q)}$ are functions of $y = v\cdot p'$. V is a low-lying vector meson, i.e., either a ρ or K^* depending on the light quark flavor quantum numbers of q and $P^{(Q)}$. Show that in the $m_{b,c}\to\infty$ limit

$$\begin{aligned} f^{(b)}(y) &= (m_b/m_c)^{1/2} f^{(c)}(y),\\ g^{(b)}(y) &= (m_c/m_b)^{1/2} g^{(c)}(y),\\ a_+^{(b)}(y) + a_-^{(b)}(y) &= (m_c/m_b)^{3/2}\big[a_+^{(c)}(y) + a_-^{(c)}(y)\big],\\ a_+^{(b)}(y) - a_-^{(b)}(y) &= (m_c/m_b)^{1/2}\big[a_+^{(c)}(y) - a_-^{(c)}(y)\big].\end{aligned}$$

Discuss how these results may be used to determine V_{ub} from data on the semileptonic decays $B\to\rho e\bar{\nu}_e$ and $D\to\rho\bar{e}\nu_e$.

4. Consider the matrix element

$$\begin{aligned}\langle V(p',\epsilon)|\bar{q}\sigma_{\mu\nu}Q|P^{(Q)}(p)\rangle &= -ig_+^{(Q)}\epsilon_{\mu\nu\lambda\sigma}\epsilon^{*\lambda}(p+p')^\sigma - ig_-^{(Q)}\epsilon_{\mu\nu\lambda\sigma}\epsilon^{*\lambda}(p-p')^\sigma\\ &\quad - ih^{(Q)}\epsilon_{\mu\nu\lambda\sigma}(p+p')^\lambda(p-p')^\sigma(\epsilon^*\cdot p).\end{aligned}$$

Show that in the $m_Q\to\infty$ limit the form factors $g_\pm^{(Q)}$ and $h^{(Q)}$ are related to those in Problem 3 by

$$\begin{aligned} g_+^{(Q)} - g_-^{(Q)} &= -m_Q g^{(Q)},\\ g_+^{(Q)} + g_-^{(Q)} &= f^{(Q)}/2m_Q + \frac{p\cdot p'}{m_Q}g^{(Q)},\\ h^{(Q)} &= -\frac{g^{(Q)}}{m_Q} + \frac{a_+^{(Q)} - a_-^{(Q)}}{2m_Q}.\end{aligned}$$

5. Verify the expressions for the $P\to\ell\bar{\nu}_e$, $\bar{B}\to De\bar{\nu}_e$, and $\bar{B}\to D^*e\bar{\nu}_e$ decay rates given in the text.

6. The fields $D_2^{*\mu\nu}$ and D_1^μ destroy the spin-two and spin-one members of the excited doublet of charmed mesons with $s_\ell = 3/2$ and positive parity. Show that

$$F_v^\mu = \frac{(1+\slashed{v})}{2}\left\{D_2^{*\mu\nu}\gamma_\nu - \sqrt{\frac{3}{2}}D_1^\nu\gamma_5\left[g_\nu^\mu - \frac{1}{3}\gamma_\nu(\gamma^\mu - v^\mu)\right]\right\},$$

satisfies

$$\slashed{v}F_v^\mu = F_v^\mu,\qquad F_v^\mu\slashed{v} = -F_v^\mu,\qquad F_v^\mu\gamma_\mu = F_v^\mu v_\mu = 0,$$

and that under heavy charm quark spin transformations

$$F_v^\mu \to D(R)_c F_v^\mu.$$

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7. Use Lorentz, parity, and time-reversal invariance to argue that the form factor decompositions of matrix elements of the weak vector and axial vector $b \to c$ currents are

$$\frac{\langle D_1(p',\epsilon)|V^\mu|\bar{B}(p)\rangle}{\sqrt{m_B m_{D_1}}} = -i f_{V_1}\epsilon^{*\mu} - i\left(f_{V_2}v^\mu + f_{V_3}v'^\mu\right)(\epsilon^*\cdot v),$$

$$\frac{\langle D_1(p',\epsilon)|A^\mu|\bar{B}(p)\rangle}{\sqrt{m_B m_{D_1}}} = f_A \epsilon^{\mu\alpha\beta\gamma}\epsilon^*_\alpha v_\beta v'_\gamma,$$

$$\frac{\langle D_2^*(p',\epsilon)|A^\mu|\bar{B}(p)\rangle}{\sqrt{m_B m_{D_2^*}}} = -i k_{A_1}\epsilon^{*\mu\alpha}v_\alpha + \left(k_{A_2}v^\mu + k_{A_3}v'^\mu\right)\epsilon^*_{\alpha\beta}v^\alpha v^\beta,$$

$$\frac{\langle D_2^*(p',\epsilon)|V^\mu|\bar{B}(p)\rangle}{\sqrt{m_B m_{D_2^*}}} = k_V \epsilon^{\mu\alpha\beta\gamma}\epsilon^*_{\alpha\sigma}v^\sigma v_\beta v'_\gamma,$$

where v' is the four velocity of the final charmed meson and v the four velocity of the $\bar{B}$ meson. Note that the D_1 polarization vector is denoted by ϵ_α while the D_2^* polarization tensor is denoted by $\epsilon_{\alpha\beta}$.

8. Show that

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}w}(\bar{B}\to D_1 e\bar{\nu}_e) = \frac{G_F^2|V_{cb}|^2 m_B^5}{48\pi^3} r_1^3\sqrt{w^2-1}\Big\{\left[(w-r_1)f_{V_1} + (w^2-1)\left(f_{V_3}+r_1 f_{V_2}\right)\right]^2 + 2\left(1-2r_1 w + r_1^2\right)\left[f_{V_1}^2 + (w^2-1)f_A^2\right]\Big\},$$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}w}(\bar{B}\to D_2^* e\bar{\nu}_e) = \frac{G_F^2|V_{cb}|^2 m_B^5}{48\pi^3} r_2^3 (w^2-1)^{3/2}\left\{\frac{2}{3}\left[(w-r_2)k_{A_1} + (w^2-1)\left(k_{A_3}+r_2 k_{A_2}\right)\right]^2 + \left[1-2r_2 w + r_2^2\right]\left[k_{A_1}^2 + (w^2-1)k_V^2\right]\right\},$$

where the form factors, which are functions of $w = v\,.\,v'$, are defined in problem 7.

9. Argue that for $B\to D_1$ and $B\to D_2^*$ matrix elements, heavy quark spin symmetry implies that one can use

$$\bar{c}_{v'}\Gamma b_v = \tau(w)\mathrm{Tr}\left\{v_\sigma \bar{F}^\sigma_{v'}\Gamma H_v^{(b)}\right\},$$

where $\tau(w)$ is a function of w, and F_v^μ was defined in Problem 6. Deduce the following expressions for the form factors

$$\begin{aligned}
\sqrt{6}\, f_A &= -(w+1)\tau, & k_V &= -\tau,\\
\sqrt{6}\, f_{V_1} &= -(1-w^2)\tau, & k_{A_1} &= -(1+w)\tau,\\
\sqrt{6}\, f_{V_2} &= -3\tau, & k_{A_2} &= 0,\\
\sqrt{6}\, f_{V_3} &= (w-2)\tau, & k_{A_3} &= 0.
\end{aligned}$$

Only the form factor f_{V_1} can contribute to the weak matrix elements at zero recoil, $w=1$. Notice that $f_{V_1}(1)=0$ for any value of $\tau(1)$. Is there a normalization condition on $\tau(1)$ from heavy quark flavor symmetry?

10. The ground-state baryons with two strange quarks and a heavy quark decay weakly, $\Omega_b \to \Omega_c^{(*)}e\bar{\nu}_e$. They occur in a $s_\ell = 1$ doublet, and the spin-1/2 and spin-3/2 members are denoted

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by Ω_Q and Ω_Q^* respectively. Show that the field

$$S_{v\mu}^{(Q)} = \left[\frac{1}{\sqrt{3}}(\gamma_\mu + v_\mu)\gamma_5 \Omega_v^{(Q)} + \Omega_{v\mu}^{*(Q)}\right]$$

transforms under heavy quark spin symmetry as

$$S_{v\mu}^{(Q)} \to D(R)_Q S_{v\mu}^{(Q)}.$$

Here $\Omega_v^{(Q)}$ is a spin-1/2 field that destroys a Ω_Q state with amplitude $u(v,s)$ and $\Omega_{v\mu}^{*(Q)}$ is a spin-3/2 field that destroys a Ω_Q^* state with amplitude $u_\mu(v,s)$. Here $u_\mu(v,s)$ is a Rarita-Schwinger spinor that satisfies $\not{v} u_\mu(v,s) = u_\mu(v,s)$, $v^\mu u_\mu(v,s) = \gamma^\mu u_\mu(v,s) = 0$. Argue that for $\Omega_Q \to \Omega_Q^{(*)}$ matrix elements heavy quark symmetry implies that

$$\bar{c}_{v'} \Gamma b_v = \mathrm{Tr} \bar{S}_{v'\mu}^{(c)} \Gamma S_{vv}^{(b)} [-g^{\mu\nu} \lambda_1(w) + v^\mu v'^\nu \lambda_2(w)].$$

Show that heavy quark flavor symmetry requires the normalization condition

$$\lambda_1(1) = 1$$

at zero recoil.

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