QUALITATIVE DYNAMICS OF THE SUN-JUPITER-SATURN SYSTEM

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ABSTRACT

The stability of the three-body problem formed by the Sun, Jupiter and Saturn is investigated using surfaces of zero velocity. The results obtained with the models of the restricted and general problems of three bodies are compared with numerical integration. The system is found to be stable in the sense that Saturn will neither interrupt the (perturbed) binary orbit of Jupiter around the Sun, nor will it escape from the system. It is shown that the known classical triple stellar systems are "more stable" than the solar system, which in turn is "more stable" than the Earth-Moon system.

INTRODUCTION

A general trend for instability of three-body systems containing masses of the same orders of magnitude was demonstrated ten years ago by Agekian (1967) and Szebehely (1967-a). These results suggested to Kuiper (1973) that the onset of instability of planetary systems may be enhanced by increasing the masses of the participating planets. In this way the method known as the K-N-S theory (Kuiper-Nacozy-Szebehely) was born and announced by Nacozy (1976). Numerical integrations using increased planetary masses were performed by Nacozy (1976), and others. Nacozy could not detect secular terms in the orbital elements of Saturn unless the masses of Jupiter and Saturn were increased by a factor (γ) of 29. When this factor was below 29 the system was stable (as found by numerical integration and as defined by the absence of secular terms). When γ was larger than 29 instability set in very soon after the beginning of the motion, as displayed by the appearance of secular terms.

This paper uses analytic qualitative methods as opposed to numerical integration to find the value of the above mentioned factor γ at which instability sets in. The raison d'être for such study is that the establishment of stability or the detection of long-period secular

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terms by numerical means is always open to questions.

ANALYSIS

First the admittedly weak model of the restricted problem is used to evaluate the effect of γ as described for instance by Szebehely (1967-b). The Jacobian constant for the orbit of Saturn is computed using the Sun and Jupiter as the primaries. In this way the mass-parameter of the restricted problem becomes $\mu = m_J/(m_\odot + m_J) = 9.539 \times 10^{-4}$, where m_J and $m_\odot$ are the masses of Jupiter and of the Sun. When the masses of Jupiter and Saturn are increased by a factor of γ we have for the new mass-parameter $\mu' = \gamma m_J/(m_\odot + \gamma m_J) \cong \gamma\mu$. The assumption is made at this point that Saturn's orbit is fixed while μ changes. In this way the Jacobian constant (C) for Saturn's orbit may be computed as a function of the mass-parameter μ' or as a function of γ. In fact, we have

$$C = 3.2516 - 0.00128\gamma .$$

On the other hand, the topology of the permissible regions of motion is controlled by the critical value of the Jacobian constant, C_{cr}, corresponding to the above introduced value of μ'. The functional relation $C_{cr} = C_{cr}(\gamma)$ is complicated but it may be approximated by

$$C_{cr} = 3.0831 + 0.00774\gamma$$

in the range of interest. The intersection of the above two straight lines corresponds to $C_{cr} = C$ and $\gamma = 18.7$. Therefore, the system is unstable (in the sense that Saturn may penetrate the Sun-Jupiter region) if $\gamma > 18.7$. On the other hand, if $\gamma < 18.7$ Saturn cannot enter the region occupied by the Sun and Jupiter.

The second model is the general problem of three bodies when orbital eccentricies as well as Saturn's effect on Jupiter and on the Sun are included in the analysis. The role of the Jacobian constant is now played by the dimensionless stability parameter $s = -c^2H/(G^2\bar{m}^5)$ which controls the topology of the zero-velocity surfaces. Here c is the angular momentum, H is the total energy, $\bar{m}$ is the average mass and G the gravitational constant. Once again, we compute the actual and the critical values of s for various values of γ and the intersection of the curves $s = s(\gamma)$ and $s_{cr} = s_{cr}(\gamma)$ will furnish the separation between stability and instability. In the range of interest we have

$$s - s_{cr} = 10^{-6}(14.83 - 1.09\gamma),$$

giving $\gamma = 13.6$ for the intersection. Therefore, instability sets in sooner (at a lower value of γ) when the (more realistic) model of the general problem of three bodies is used, while the model of the restricted problem gives more tolerance.

The computation of the stability parameter is described by Szebehely and McKenzie (1977) with additional details.

Similar analysis may be used to study the stability of classical triple stellar systems. For observed systems the measure of stability is given by $S = (s - s_{cr})/s_{cr}$ and it is found to be of the order of one. The same measure of stability for the above described model of the solar system ($\gamma = 1$) is $S = 3.6 \times 10^{-2}$, consequently, the known triple stellar systems are "more stable" than the solar system.

The Sun-Earth-Moon system is stable according to Hill's (1878) computation as well as according to the restricted problem (Szebehely, 1967-b). The measure of stability found by using the model of the restricted problem for the moon's orbit is $S = (C - C_{cr})/C_{cr} \cong 10^{-4}$, consequently, the moon's orbit is much "less stable" than the solar system. Corresponding values for the moon's stability, using the general problem are not available yet, but the stability is expected to be reduced because of the eccentricities of the orbits and because of the presence of general three-body effects. The hierarchy of stability according to these results is: triple stellar systems, planetary systems and satellites, in order of decreasing stability.

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REFERENCES

Agekian, T.A., and Anosova, Z.P., (1967), Astronomical Zhurnal, **44**, 1261.

Hill, G., (1878), Am. J. Math., **1**, 5, 129, 245.

Nacozy, P., (1976), "On the Stability of the Solar System", Astron. J., **81**, 787.

Szebehely, V., (1967-a), Proc. U.S. Nat. Acad. Sc., **58**, 60.

Szebehely, V., (1967-b), Theory of Orbits, Academic Press, New York, NY.

Szebehely, V., and McKenzie, R., (1977), "Stability of Planetary Systems with Bifurcation Theory", Astron. J. **82**, 79.