PART I

EXTINCTION

https://doi.org/10.1017/S0074180900053961 Published online by Cambridge University Press

SOME SCATTERING PROBLEMS OF INTERSTELLAR GRAINS*

J. MAYO GREENBERG

State University of New York at Albany

and

Dudley Observatory

Abstract. Some general effects of size, shape, and material on the scattering properties of small particles are considered with a view to determining their relative importance to interstellar dust interpreatations.

1. Introduction

Although this was originally intended to be a broad introduction to the symposium it seemed more useful, after seeing the excellent review papers as well as the contributed papers, to limit my discussion to a few basic scattering problems. A substantial portion of this paper on the nonspherical particle scattering consists of previously unpublished results which have not yet been fully exploited toward understanding the interstellar dust problem but which are clearly relevant and ultimately must be applied.

The most general way of defining the scattering problem is in terms of size, shape, and index of refraction. The index of refraction may be simple as in the case of an isotropic homogeneous particle or it may be complicated as in the case of such inhomogeneities as occur when the particle accretes a mantle of one material on a core of another. A somewhat more subtle physical phenomenon is involved if the particles are sufficiently small that bulk optical properties are no longer applicable. Very little has been done on this latter problem as regards conducting particles. In any case it is a problem which, as we shall see in the discussion of the far ultraviolet extinction, may be of considerable importance and deserve a great deal of attention.

Disregarding size effects, the character of the index of refraction is determined primarily by the chemical constituents of the grains. However, even here there may be significant modifications produced by the physical state of the particle; e.g., temperature, radiation damage, frozen free radicals.

The interstellar particle shapes are obviously mostly nonspherical as evidenced by the amount of interstellar polarization relative to the interstellar extinction.

A full definition of the interstellar grain optics involves therefore, a knowledge not only of the composition, size and shape of the particles but also their physical state.

2. Indices of Refraction

Current evidence leads us to consider silicates, ices, and carbon or combinations of these as the principal grain ingredients.

* Work supported in part by NASA grant #NGR-33-011-043

Greenberg and Van de Hulst (eds.), Interstellar Dust and Related Topics, 3–9.

The most direct evidence for the silicates and to a certain extent the ices comes from the infrared spectral characteristics of the extinction curve. The limited direct comparison which can be made or the relative amount of silicates and ices is based on the observations of Gillett and Forrest (1973). From their curve the absorption at 3.1 μ is very close to that at 10 μ. It is normal to attribute the 3.1 μ absorption to H_2O ice and the 10 μ absorption to a silicate. The mass of absorbing material may then be estimated for spherical particles assumed to be small enough relative to the wavelength so that the Rayleigh approximation, $x = 2\pi/\lambda \ll 1$ is valid but not so small that the bulk refractive index does not apply. The expression for the optical depths is

$$\tau = \frac{18\pi}{\lambda} V \frac{y}{(\varepsilon+2)^2 + y^2}, \tag{1}$$

where V is the total volume of the material $\varepsilon = m'^2 - m''^2$, $y = 2\,m'm''$, m' = real part of refractive index, m'' = imaginary part.

For $\tau_{ice} = \tau_{sil}$ we get

$$\frac{V_{ice}}{V_{sil}} = \frac{3.1}{10} \frac{y_{sil}/[(\varepsilon_{sil}+2)^2 + y_{sil}^2]}{y_{ice}/[(\varepsilon_{ice}+2)^2 + y_{ice}^2]} = 0.21, \tag{2}$$

where we have used the values $m_{ice} = 1.375–0.815$ (Bertie *et al.*, 1969) and $m_{sil} = 1.7–0.71\,i$ (Greenberg, 1972, as deduced from Launer, 1952) as the respective complex indices for ice and silicate at their absorption maxima. The ratio of volumes given in Equation (2) leads to a *mass* ratio of silicate to H_2O ice of 10:1 which is somewhat larger than the mass ratio of 4:1 derived by Gillett and Forrest, this difference being probably due to the fact that we have used an updated value of m for ice which gives greater absorptivity. We note that in the dirty ice model made up of water, methane, and ammonia, the above results really imply a somewhat larger relative volume of dirty ice. If we roughly assume that O, C, and N ices contribute relative volumes to the total according to their cosmic abundance the value 0.21 should be raised by a factor of

$$\frac{[O]+[C]+[N]}{[O]} = 1.5\,.$$

If further we follow the argument that photolysis by ultraviolet and other radiation in space may be shifting the 3.07 band (Greenberg, 1972) it would appear that there is a substantial amount of ices in interstellar space. A resonable assumption is that the ice forms by accretion from the interstellar gas as a mantle on a silicate core, thus leading to inhomogeneous particles.

On the basis of the infrared observation, we are thus led to consider such dielectric particles as silicates, ices, and inhomogeneous combinations of these.

The evidence for other materials is somewhat less direct. In particular, in Section 4, we shall indicate the most convincing basis for consideration of graphite as an interstellar grain constituent.

3. Shape Effects on Mass Estimates

The estimate of ice and silicate masses is subject to errors, not only in the assumption of bulk optical properties but also in the shape assumed. For the former there is no definite answer but for the latter it may readily be shown that spheroidal particles may be 10–75% more efficient *absorbers* in Rayleigh approximation thus implying that mass estimates based on spherical shape are too large. It should also be kept in mind that estimates of visual extinction based on material mass (and hence volume) should properly include consideration of shape effects.

Let us compare prolate and oblate spheroids to spheres. Other elongated and flat particles produce the same effects. Assuming equal volumes of spheres and non-spheres producing the given IR absorption we have

prolate
or
oblate

$$n_1 \tfrac{4}{3}\pi a^3 = n_2 \tfrac{4}{3}\pi B^2 A, \tag{3}$$

where n_1 and n_2 are the spatial number densities of spheres and spheroids respectively and where A is taken as the rotation semi-axis of the spheroids.

For needles $A/B \to \infty$ and for plates $A/B \to 0$. In these limits the ratios of sphere to spheroid extinction are

$$\text{prolate:} \quad \frac{4}{\pi}\,\frac{B}{a} \tag{4}$$

$$\text{oblate:} \quad 2A/a\,.$$

These ratios clearly depend on the relative 'size' chosen for the spherical and non-spherical particles. Equal volumes (of individual particles) and equal thcikness produce quite different effects. There is no obvious a priori way of making a choice other than by matching the observed extinction curve with models of each particle type and thus defining the appropriate relative sizes based on shape. The two-fold dimension dominates and it turns out that less than 20% difference is formed between the radii of very long cylinders and of spheres used to produce similar extinction curves (Greenberg, 1968).

4. Particle Sizes

The general shapes of the curves for the wavelength dependence of interstellar extinction and polarization and the choice of index of refraction are the means of defining typical particle sizes. The shape of the extinction curve in a particular wavelength region gives information primarily about the particles whose size is in the same region. This is due to the universal character of the dependence of extinction by individual particles on the dimensionless parameters $x = 2\pi a/\lambda$ and/or $\rho = 2x(m' - 1)$. This has been demonstrated numerous times in the literature.

We should examine the representative extinction curve shown in Figure 1 as it has

https://doi.org/10.1017/S0074180900053961 Published online by Cambridge University Press

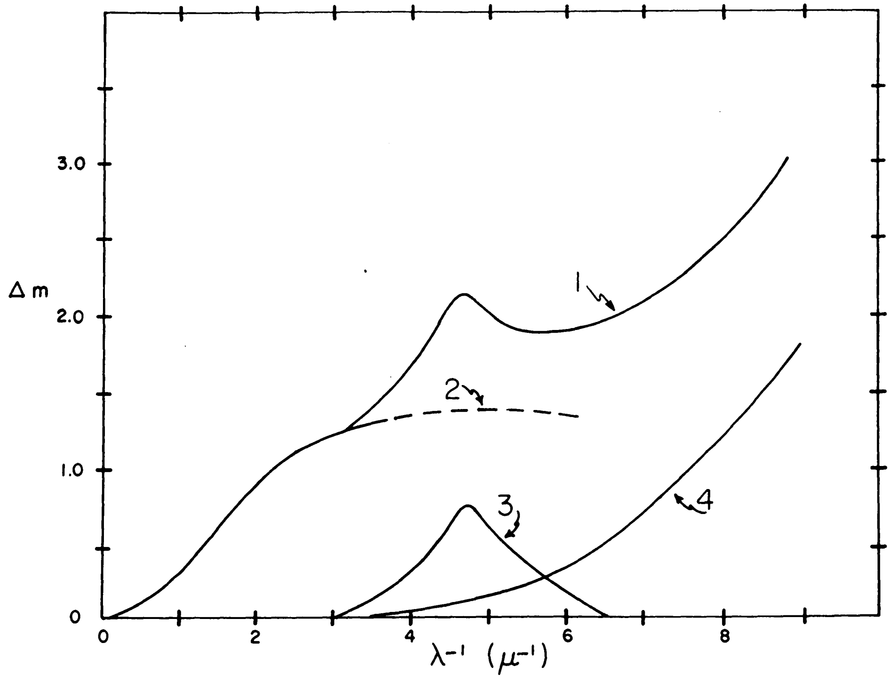

Fig. 1. Representative extinction curve and schematic separation into several contributions. Curves are labeled as follows: (1) Typical OAO extinction curve, (2) Dashed extension showing contribution by classical sized particles (as discussed in text), (3) Contribution of absorption in the 0.22μ band, (4) Contribution by very small particles.

been observed by Bless and Savage (1972). The 'visual' portion ($1 \leq \lambda^{-1} \leq 3\mu^{-1}$) is characterized by the classical particles whose size (radius) is $\bar{a}_{\text{ice}} = 0.165\ \mu$ radius for dirty ice and $\bar{a}_{\text{sii}} = 0.082$ for silicates. Core mantle particles with silicate core of about 0.05 μ radius and ice mantles of about 0.10 μ would probably also be satisfactory. In these dielectric particles it is the parameter ρ which seems to apply. Much work has been done on the portion labeled '2'. The part labeled '3' is often attributed to graphite although other suggestions involving silicates (Huffman and Stapp, 1971) have been made. The particle sizes associated with either of these models are much smaller than the classical sizes. If the contribution to the extinction labeled '4' is due to solid particles, their sizes would be less than 100 Å (Greenberg, 1973). An anomaly with respect to the far ultraviolet albedo of these grains has not yet been resolved (Witt and Lillie, 1971).

In order to account for the observed extinction in terms of solid particles we must therefore include particles from smaller than 0.01 μ to as large or larger than 0.15 μ. Since the mass contribution to the interstellar medium is proportional to the size (all other factors being equal) we see that even for equal contributions to the total extinction in the far ultraviolet the small particles add only the order of 10% to the total interstellar dust material.

We have implicitly in all the above, assumed that the small particles react optically in the same way as their classical counterparts. There are a number of reasons why this may not be the case. In the next section we consider one of these.

5. Small Particle Temperatures

It has been noted (Greenberg, 1968) that the definition of the temperature of very small particles is subject to fluctuations which may become sufficiently large that the 'average' temperature loses significance.

We first calculate the grain temperature as if it were properly defineable and then compare the likely fluctuations in the particle's heat content with the average value which is given by the temperature.

Radiation equilibrium may be described by the equation

$$\bar{\varepsilon}_{ab} \int_0^\infty R(\lambda)\, \mathrm{d}\lambda = \bar{\varepsilon}_{em} \int_0^\infty B(\lambda, T_g)\, \mathrm{d}\lambda\,, \tag{5}$$

where $\bar{\varepsilon}_{ab}$ and $\bar{\varepsilon}_{em}$ are the average efficiencies for grain absorption from the interstellar radiation field $R(\lambda)$ and grain emission at temperature T_g respectively. For $\bar{\varepsilon}_{em} = \varepsilon_{\text{eff}} = 33.3\ aT_g$ (the maximum microwave emissivity as defined in Greenberg (1971), we find a lower bound on the small particle temperature to be given by

$$\bar{\varepsilon}_{ab} W T_R^4 = \varepsilon_{\text{eff}} T_g^4 = 33.3 a T_g^5\,, \tag{6}$$

where we have let the interstellar field be at a temperature T_R diluted by the factor W.

The values of $\bar{\varepsilon}_{ab}$ and $\bar{\varepsilon}_{em}$ are strongly size dependent. For example, we may note that $\bar{\varepsilon}_{ab} = 0.1$ for a 0.1 μ particle and is $\bar{\varepsilon}_{ab} = 1.0$ for a 1.0 μ particle (Greenberg, 1968). For $W = 10^{-14}$, $T_R = 10000°$, $a = 10^{-5}$ cm, $\bar{\varepsilon}_{ab} = 0.1$ we get $T_g = 8°$. Noting that $\bar{\varepsilon}_{ab}$ for a 10^{-6} cm particle is certainly less than that for a 10^{-5} cm particle we may immediately note that the temperature of a 10^{-6} cm particle would be very close to that of a 10^{-5} cm particle. This could have been expected from extrapolation of the curve showing the variation of grain temperature with size (Greenberg, 1968). However, the temperature fluctuations will be substantial.

Following standard procedures (Kittel, 1956) we find the heat content of a low temperature particle to be

$$U = (3\pi^4/5)\, N(kT_g)(T_g/\Theta)^3\,,$$

where Θ is the Debye temperature and N is the total number of molecules in the grain. Using a representative value of $\Theta = 300$ and obtaining N by dividing the volume of the grain by the volume per molecule assumed to be $\sim 8 \times 10^{-24}$ cm^3 we get $U = 1.55 \times 10^{-12}$ ergs or about 1 eV. Thus a 5eV photon ($\lambda = 2480$ Å) would give rise to a grain internal energy of about five times the average and the instantaneous grain 'temperature' would be 15°. If the accretion of condensible atoms is a sensitive function of grain temperature, frequent temperature spikes would inhibit grain mantle growth. One possible consequence could be that if we were to inject a size spectrum of silicate particles into interstellar space, there could be a size below which dirty ice mantles could not accrete so that one would find a mixture or very small 'pure' silicate particle with a distribution of silicate core-ice mantle particles. This would be a possible simultaneous way of accounting for the classical portion '2' of the extinction curve by core-mantle particles and the ultraviolet portion, '4', by small silicate particles. In dark regions of interstellar space we might than expect that all particles would be at a sufficiently low temperature and/or subject to so few high energy photons that they could accrete the condensible gas material and that in such regions the ultraviolet rise in the extinction should disappear. It is rather suggestive that this combination of core-mantle and small bare particles is the most consistent with silicate absorption observations and cosmic abundance arguments.

Moesta (private communication) has suggested another mechanism which would lead to lack of accretion on very small particles.

6. Nonspherical Particle Scattering

This section is inteded to show that we are now capable of performing essentially precise theoretical calculations on the scattering by smooth nonspherical particles up to and beyond the resonance region. The details are much too cumbersome to report here but the results as obtained by Reilly (1969) are convincing evidence on the reliability of the method. It should be mentioned that although we limit ourselves to presenting only extinction efficiencies, the method produces *complete* scattering information analogous to Mie theory.

Extinction efficiencies for prolate spheroids of 2:1 ratio and refractive index $m = 1.33-0.05\,i$ are shown in Figure 2 for three orthogonal orientations. Experimental results from the microwave scattering laboratory are seen to be quite reliable in the size range. For comparison are shown the E and H extinction efficiencies of perfectly

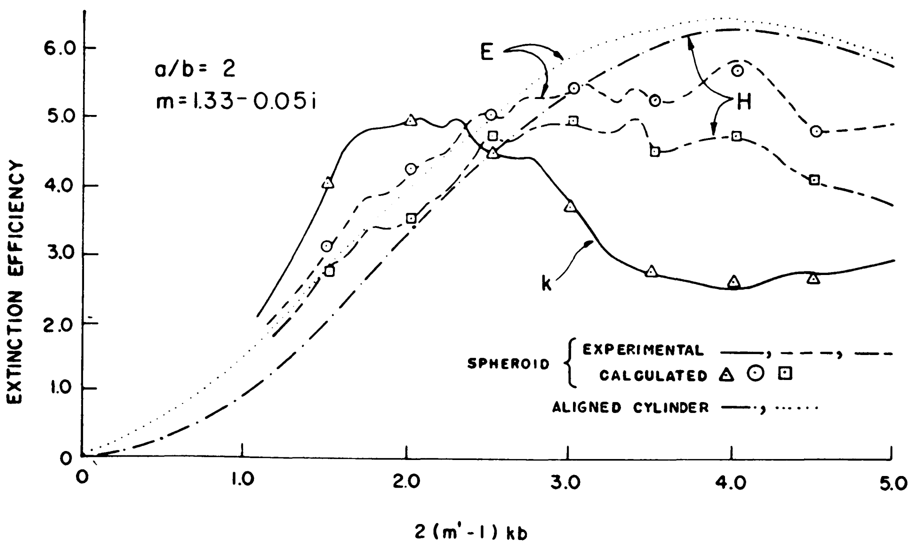

Fig. 2. Theoretical and experimental extinction curves for prolate spheroids and cylinders.

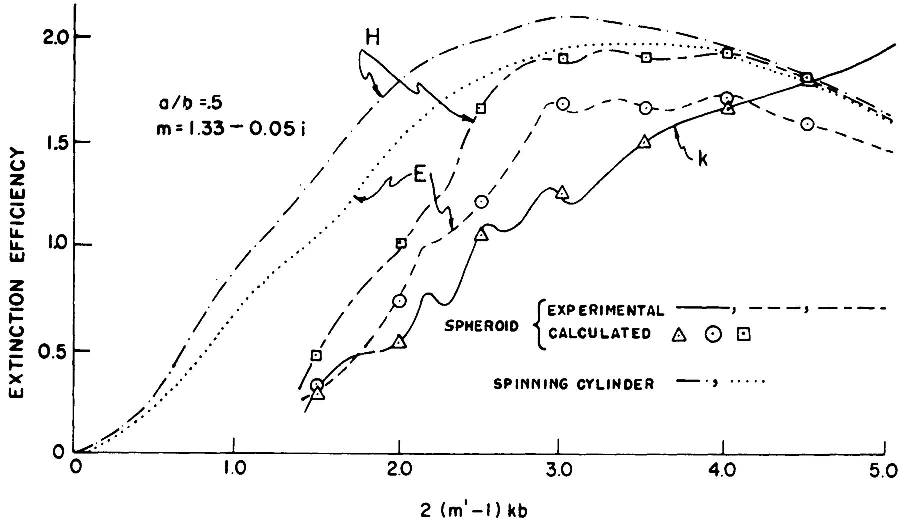

Fig. 3. Theoretical and experimental extinction curves for oblate spheroids and spinning cylinders.

aligned infinite cylinders normalized to correspond to the same asymptotic ($x \to \infty$) value of 4 for the E and H prolate spheroids. Although the results differ in detail, it appears that semi-quantitative inferences based on cylinders are realistic. In particular, the degree of polarization is not very different between the finite spheroid and the infinite cylinder.

In Figure 3 are shown some new theoretical results for 1:2 oblate spheroids. Again comparison is made with experimental results. We have calculated the average extinction from a set of cylinders spinning about an axis as a reasonable basis for comparison with the oblate spheroid. For small sizes the results are quite disparate but in the resonance region there is some similarity. Of particular note is that the degree of polarization by the spinning cylinders is both comparable with the oblate spheroid as well as the perfectly aligned cylinders.

References

Bertie, J. E., Labbé, A. J., and Whalley, E.: 1969, *Chem. Phys.* **50**, 4501.

Bless, R. C. and Savage, R. D.: 1972, *Astrophys. J.* **171**, 293.

Gillett, F. C. and Forrest, W. J.: 1973, *Astrophys. J.* **179**, 483.

Greenberg, J. M.: 1968, in B. M. Middlehurst and L. H. Alter (eds.), *Nebulae and Interstellar Matter*, Univ. of Chicago Press, Chicago, p. 221.

Greenberg, J. M.: 1971, *Astron Astrophys.* **12**, 240.

Greenberg, J. M.: 1972, *J. Colloid Interface Sci.* **39**, 513.

Greenberg, J. M.: 1973, in M. A. Gordon and L. E. Snyder (eds.), *Molecules in the Galactic Environment*, J. Wiley & Sons, p. 94.

Huffman, D. R. and Stapp, J. L.: 1971, *Nature Phys. Sci.* **229**, 45.

Launer, P. J.: 1952, *Am. Minerologist* **37**, 764.

Reilly, E. D., Jr.: 1969, Thesis, R.P.I.

Witt, A. N. and Lillie, C. F.: 1971, *OAO Symposium*, Amherst, Mass.