Appendix B

Weight factors for $SU(N)_c$

B.1 Definition

The generators T_a of the $SU(N)_c$ Lie algebra obey the commutation relation:

$$[T_a, T_b] = i f_{abc} T_c \tag{B.1}$$

and the trace properties:

$$Tr\ T_a = 0\ . \tag{B.2}$$

f_{abc} are constants which are *real* and totally antisymmetric and normalized as:

$$f_{abc} f_{dbc} = N \delta_{ad}\ . \tag{B.3}$$

B.2 Adjoint representation of the gluon fields

In this representation, one has:

$$(T_a)_{bc} = -i f_{abc}\ , \tag{B.4}$$

with the properties:

$$\begin{aligned} f_{abe} f_{cde} &= \frac{2}{N}[\delta_{ac}\delta_{bd} - \delta_{ad}\delta_{bc}] + d_{ace}d_{dbe} - d_{ade}d_{bce}\ , \\ & f_{abe}d_{cde} + f_{ace}d_{dbe} + f_{ade}d_{bce} = 0\ , \end{aligned} \tag{B.5}$$

where d_{abc} is a real and totally symmetric tensor:

$$\begin{aligned} d_{abb} &= 0\ , \\ d_{abc}d_{dbc} &= (N - 4/N)\,\delta_{ad}\ . \end{aligned} \tag{B.6}$$

In this representation, the trace properties are:

$$\begin{aligned} &Tr\ T_a T_b = N\delta_{ab}\ , \\ &Tr\ T_a T_b T_c = \frac{i}{2} N\delta_{ab}\ , \\ &Tr\ T_a T_b T_c T_d = \delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} + \frac{N}{4}\left(d_{abe}d_{cde} - d_{ace}d_{dbe} + d_{ade}d_{bce}\right)\ . \end{aligned} \tag{B.7}$$

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B.3 Fundamental representation of the quark fields

In this case:

$$T_a = \frac{1}{2}\lambda_a \; , \tag{B.8}$$

with the properties:

$$\begin{aligned} [\lambda_a, \lambda_b] &= 2i f_{abc}\lambda_c \; , \\ \{\lambda_a, \lambda_b\} &= \frac{4}{n}\delta_{ab} + 2d_{abc}\lambda_c \; , \\ \lambda_a\lambda_b &= \frac{2}{N}\delta_{ab} + d_{abc}\lambda_c + i f_{abc}\lambda_c \; . \end{aligned} \tag{B.9}$$

The trace properties are:

$$\begin{aligned} Tr\;\lambda_a &= 0 \\ Tr\;\lambda_a\lambda_b &= 2\delta_{ab} \\ Tr\;\lambda_a\lambda_b\lambda_c &= 2(d_{abc} + i f_{abc}) \\ Tr\;\lambda_a\lambda_b\lambda_c\lambda_d &= \frac{4}{N}(\delta_{ab}\delta_{cd} - \delta_{ac}\delta_{bd} + \delta_{ad}\delta_{bc}) \\ &\quad + 2(d_{abe}d_{cde} - d_{ace}d_{abe} + d_{ade}d_{bce}) \\ &\quad + 2i(d_{abe}f_{cde} - d_{ace}f_{abe} + d_{ade}f_{bce}) \; . \end{aligned} \tag{B.10}$$

Some other useful relations are:

$$\begin{aligned} (\lambda_a)_{\alpha\beta}(\lambda_a)_{\gamma\delta} &= 2\left(\delta_{\alpha\delta}\delta_{\beta\gamma} - \frac{1}{N}\delta_{\alpha\beta}\delta_{\gamma\delta}\right) \\ &= \frac{2(N^2-1)}{N^2}\delta_{\alpha\delta}\delta_{\beta\gamma} - \frac{1}{N}(\lambda_a)_{\alpha\beta}(\lambda_a)_{\gamma\delta} \; , \\ (\lambda_a)_{\alpha\beta}(\lambda_a)_{\beta\gamma} &= 4\left(C_2(R) \equiv \frac{N^2-1}{2N}\right)\delta_{\alpha\gamma} \; , \\ (\lambda_b\lambda_a\lambda_b)_{\alpha\beta} &= -\frac{2}{N}(\lambda_a)_{\alpha\beta} \; , \\ (\lambda_a\lambda_b)_{\alpha\beta}(T_b)_{ca} &= N(\lambda_c)_{\alpha\beta} \; . \end{aligned} \tag{B.11}$$

In the adjoint representation:

$$(T_a)_{bc}(T_a)_{cd} = (C_2(G) \equiv N)\,\delta_{bd} \; . \tag{B.12}$$

B.4 The case of $SU(3)_c$

In this case, one can write explicitly:

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \quad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

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$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} . \tag{B.13}$$

Therefore:

$$\begin{aligned} f_{123} &= +1 \\ f_{147} = f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} &= \frac{1}{2} , \\ f_{458} = f_{678} &= \frac{\sqrt{3}}{2} , \end{aligned} \tag{B.14}$$

and:

$$\begin{aligned} d_{118} = d_{228} = d_{338} = d_{888} &= \frac{1}{\sqrt{3}} \\ d_{146} = d_{157} = -d_{247} = d_{256} = d_{344} = d_{355} = -d_{366} = -d_{377} &= \frac{1}{2} , \\ d_{448} = d_{558} = d_{668} = d_{778} &= -\frac{1}{2\sqrt{3}} . \end{aligned} \tag{B.15}$$

The other components which cannot be obtained by permutation of indices of the above ones are zero.