EFFECTS OF CONVECTIVE OVERSHOOTING, MASS LOSS (AND CHEMICAL COMPOSITION) ACROSS THE HR DIAGRAM

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ABSTRACT. The far reaching consequences of convective overshooting during the core H and He-burning phases of stars in the mass range 1.3 M⊙ to 100 M⊙ are discussed. In addition to this, the effects of mass loss in luminous stars of all spectral types, and in the red giant and asymptotic giant branch stars are briefly outlined. Furthermore, the effects of the novel $^{12}C(\alpha\ \gamma)^{16}O$ reaction rate are also illustrated. The main purpose of this review resides however in lending convincing support to the idea that convective cores of real stars are greater than commonly supposed in classic models. To this aim, several observational embarrassments that could not be explained by classic models are reanalyzed in the light of the new ones. Since a much better agreement between theory and observations is now possible, we are inclined to conclude that convective overshooting may be of paramount importance in stellar structure theories and that convective cores in real stars ought to be larger by approximately one pressure scale height than predicted by classic models.

INTRODUCTION

The amount of observational data available today on star clusters and associations of different ages and chemical compositions makes it possible to study stellar evolution from an empirical point of view. Even if the general properties of stellar evolution are known from long time, still it has become evident over the recent years that many points of contradiction between classic theory and current observation exist demanding a deep revision of stellar evolution. In this paper we will limit ourselves to discuss those observational facts that led us to revise the current models of massive and intermediate mass stars. Populous young clusters of the Magellanic Clouds (LMC and SMC) are particularly suited to this purpose as they are sufficiently populated throughout the various evolutionary stages, thus allowing us to compare them safely with theoretical predictions, even for the very short lived evolutionary phases. On the contrary, this is not easily feasible with galactic open clusters and associations as they contain much fewer stars.Therefore, the above comparison can be made only by collecting data for many individual clusters and by deriving composite HR diagrams. Since many review papers exist describing the properties of massive and intermediate mass stars (Chiosi 1982a,b; Maeder

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1984; Chiosi and Maeder 1985; Iben 1974; Iben and Renzini 1983,1984), we will not go into any detail relative to classic evolutionary models. On the contrary, we will focus on four main lines of work, which have (or potentially may) deeply changed (change) the classic scenario. In fact, over the past years, our appreciation of convective overshooting, mass loss by stellar wind, nuclear reaction rates ($^{12}C(\alpha\gamma)^{16}O$ in particular), and true stellar opacity (more precisely the one due to ionization of CNO and heavier elements in the middle temperature regions, see below) has been changed either by upgraded observational information, laboratory experiments and/or theoretical considerations. It goes without saying that not all of the above physical processes are known with the same degree of confidence. In fact, while the occurrence of mass loss is indicated by observations in spite of the uncertainty still affecting the mass loss rates, only very indirect arguments can be put forward to lend support to the existence of convective overshooting. Furthermore, if the novel rate for the $^{12}C(\alpha\ \gamma)^{16}O$ reaction is supported by laboratory measurements, modification to current opacities has not yet received widespread acceptance nor clearcut confirmation or disproval by theoretical calculations due to the complexity of such a task. The evolutionary scenarios developed insofar, in which one or more of the above physical processes have been incorporated, are reflective of the underlying uncertainty. In fact, while the effects of mass loss and new $^{12}C(\alpha\ \gamma)^{16}O$ have been thoroughly investigated, those of convective overshooting all over the life of a star now begin to be assessed, those of varied opacity are still in an exploratory stage. In the following, we will mainly concentrate on convective overshooting, firstly describing the far reaching effects of this deeply seated phenomenon, and secondly searching as many as possible observational facts which, discrepant with classic models, may now be interpreted by the new ones. The great advantage shown by the new models over the old ones will be taken as strongly indicating the existence of convective overshooting in real stars. Furthermore, since many important details of this phenomenon are still poorly known, this way of proceeding will implicitly tell us more about the physical processes in which convective overshooting roots. Since the effects of opacities different from the standard ones have been occasionally investigated in the domain of massive and intermediate mass stars, they will be shortly summarized without emphasizing the possible implications. The plan for the remainder of this paper is as follows. In section I, we concisely review several observational facts that cannot be easily interpreted by classic models. In section II, we discuss the fundaments of the four physical ingredients of model construction we have been referring to insofar. In section III we present the main results for models with convective overshooting, new $^{12}C(\alpha\ \gamma)^{16}O$ reaction rate and mass loss by stellar wind (when appropriate), all over the mass range in which convective overshooting may be effective. Section IV applies those results to massive stars and evidenciates those points of uncertainty that still exist with the new models. Section V deals with several important consequences of the new models in the range of intermediate mass stars and in particular it discusses the capability of greatly improving upon the observational embarrassments presented in section I. Section VI reviews some preliminary results obtained with modified opacities. Finally, some concluding remarks are drawn in section VII.

1. SHADES OF THE OBSERVATIONAL SCENARIO

From the large body of literature dealing with observations of star clusters in our own galaxy and nearby galaxies (LMC and SMC), we have selected the following points, which in our opinion cannot be reasonably explained by common theories of stellar evolution.

1.1 Young Luminous (massive) Stars

The catalogue of all known supergiants, O type stars and less luminous B type stars in our galaxy with MK spectral types and luminosity classes compiled by Humphreys and McElroy (1984) provides the most extended source of data for galactic luminous stars. Since WR stars are commonly understood as the descendents of O type stars via the mechanism of mass loss by stellar wind (see Chiosi 1982a for a recent review of the subject), before comparing stellar counts with theoretical predictions, the Humphrey and McElroy catalogue has to be complemented by the list of known galactic WR stars (van der Hucht et al 1981). Upon transformation of Mv's and spectral types into Mb's and Teff's by means of a given Sp:Teff:BC scale, several important features of the resulting composite HR diagram are soon evident. As they have been amply discussed by Humphreys (1982), Chiosi (1982b) and Chiosi and Maeder (1985), they will not be illustrated here. I shall rather begin with a few comments of general interest and then concentrate on a result of stellar counts, which has driven most of the theoretical work done in this context. First of all, many of the features shown by that composite HR diagram, in particular the location of O type stars and related apparent decline of the maximum luminosity reached by these stars with decreasing Teff, depend od the adopted Sp:Teff:BC scale. Humphreys widely used the so-called "hot scale of Teff", which, if on one hand has received widespread consensus, on the other hand amplifies the above features. Other scales of Teff are known to exist, which associating a cooler Teff to a given spectral type make the above trend less pronounced. Secondly, the question may arise whether the Teff resulting from evolutionary computations is comparable with Teff given by observation. Strong stellar winds may in fact produce a pseudo photosphere in the flow itself, thus indicating a Teff which may be significantly cooler than the one derived from hydrostatic atmospheres (de Loore et al 1982; Bertelli et al 1984). Thirdly, the catalogues of supergiant and O-type stars suffer from a certain degree of incompleteness which is difficult to assess. This may be particularly severe for the earliest O type stars for which the bolometric corrections are the greatest. Somewhat related to this, there is another intricacy which makes the comparison with theory even more complicated. Rough stellar counts per spectral type and given luminosity interval indicate that the star frequency distribution seems not to mimic the distribution of relative lifetimes one would expect from models. It appears as there is a deficiency of stars near the zero age main sequence. Is this indicating that stellar models are in error or that the majority of O type stars are already evolved ? (see also Vanbeveren 1986). The deficiency of O type stars seems to begin at Mb < -8 or equivalently for masses greater than about 30 M$_\odot$, see for instance the HR diagram of Fig 1 in Humphreys and McElroy (1984). We will touch upon this point later on. Despite of those uncertainties, stellar counts by Meylan and Maeder (1982) and Bertelli et al (1984) have indicated an excess of A to G stars and the

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the number of core H-burning stars (main sequence band) is lower than expected. In fact, while about 10% of the total lifetime of a star is spent outside the main sequence band, the observations indicate that some 40% of the stars fall outside this region. To illustrate the point, we present below the stellar counts performed by Bertelli et al (1984) for stars of the Humphreys (1978) and Garmany et al (1982) catalogues falling in the luminosity range $-7 > M_b > -9$. WR stars pertinent to that luminosity interval were also included. These are from van der Hucht et al (1981). It must be however recalled that the bolometric luminosity of WR stars is highly uncertain thus somewhat weakening Bertelli's et al (1984) conclusions. The different grouping of spectral types given in the first column of Table 1 indicates the maximum extension of the main sequence band for models evolved at constant mass and in presence of mass loss by stellar wind.

Table 1 (From Bertelli et al 1984)

Sp	O	B	A	F	G	K	M	WR	Note
N	280	201	10	4	1	1	29	20	a
N/Nt	0.51	0.37	0.02	0.01	~0	~0	0.05	0.04	
a) models evolved at constant mass									
Sp	O-B0.5	B1-B9	A	F	G	K	M	WR	Note
N	401	80	10	4	1	1	29	20	b
N/Nt	0.73	0.15	0.02	0.01	~0	~0	0.05	0.04	
b) models evolved with mass loss by stellar wind (Chiosi et al 1978)									

It can be easily recognized that in order to reconcile standard theory with observation and considering only the most favourable case (models with mass loss) the number of stars in the spectral range from O to B0.5 should be increased by more than a factor of two. In this case effects on Teff caused by pseudo photospheres in the flow are less of a problem due to the moderate rates of mass loss pertinent to stars in that range of luminosity. If we apply the arguments of Vanbeveren (1986) to those stars, it would imply that more than 50% of the core H-burning lifetime is spent by those stars in a so-called phase of invisibility. Such a possibility was already suggested in a different context by Garmany et al (1982). Since the core H-burning lifetime of a typical 20 $M_\odot$ star is about 7×10^6 yr, the invisible phase should last a few million years. Although the time scale to remove the parent cloud left over by a star forming event is not known very well (Appenzeller 1980), hardly it can be as high as required. It may well be that a significant fraction of massive stars potentially are still in this phase and therefore not yet detectable as O type stars (infrared emitters ?), but it is likely to be less than 50 %. A plausible guess may be about 10 to 20 % (Garmany et al 1982) at least in the range of luminosity (mass) considered. A possible way out of the dilemma is to suggest that the main sequence band is actually more extended than indicated by current models even in occurrence of mass loss. Those arguments spurred the consideration of other processes such as convective overshooting and/or varied opacity (Bressan et al 1981; Bertelli et al 1984). Both concur to spread the main sequence band toward lower Teff's. However, as massive stars turned out to be a poor laboratory for testing the real occurrence of the above mentioned physical implementations, the attention has been addressed to stars of lower mass, which also presented many controversial aspects.

1.2 Intermediate Mass Stars

The main sources of observational data for this mass range are the study by Mermilliod (1981) of a sample of galactic open clusters, and the more recent analysis by Roth (1984). As for LMC clusters many different sources have been considered which will be quoted whenever appropriate. Out of the available material and previous studies on the same subject, we have selected the following points of controversy. They are briefly summarized below in order of increasing complexity.

i) Main sequence widening. Similarly to massive stars, also the main sequence of galactic open clusters in the age range of Pleiades to Hyades appears to be wider than expected (Maeder and Mermilliod 1981). The same property is also shown by LMC clusters like NGC 1866 and others of similar type (Becker and Mathews 1983).

ii) Morphology of HR diagrams. Furthermore, the overall morphology of the above clusters cannot be easily interpreted. In fact, looking at the composite HR diagram of Mermilliod (1981) for galactic open clusters it turns up that very few stars are located in the middle of the Hertzsprung gap with the exception of a few Cepheids and/or composite spectroscopic binaries (gK+A). Furthermore, one also notices that red giants are rather grouped showing two distinct trends in the pattern of red giant concentration. Red giants of relatively aged clusters (from Hyades to NGC 6475) occur at about the same colour - $(B-V)_o = 1.0$ - while the absolute magnitude becomes higher. On the contrary, red giants of younger clusters become redder and redder as the absolute magnitudes get higher and the luminosity class gradually changes. It is clear that the relative number of star in different areas of the HR diagram (main sequence, blue and red giants, asymptotic giant branch stars) are related to the lifetime of the underliying phase. Even if the correspondent evolutionary phases, namely core H-burning, core He-burning (in the loop and along the Hayashi line) and double shell stages beyond central He-burning are well assessed, the precise relative duration of these stages has never been carefully tested on observational basis. The only study we are aware of is by Lindoff (1969), who analyzing 108 galactic open clusters found that giant star (blue and red) lifetime versus main sequence lifetime indicated by the observational data was much shorter (by about a factor of 2 to 3) than predicted by those days evolutionary models. Amazingly enough, the disagreement is still there even with more recent models (Becker 1981). In addition to this, the HR diagram seems to also to indicate that in galactic environment (chemical composition) extended loops during the core He-burning phase are not likely to occur or that the time spent in the "blue loop" is much shorter than the time spent along the Hayashi line. On the contrary, the existence of blue loops in intermediate age clusters of LMC is well documented by their HR diagram (NGC 1866 as a prototype). However, even in this case, the ratio of blue to red giants is not entirely compatible with classic models. Observations indicate 1, while theory yields 0.3 (Becker and Mathews 1983). Variations in the chemical composition parameters seem insufficient to remove the disagreement.

iii) Lack of bright AGB Stars. The remarkable absence in NGC 1866 and other similar clusters of LMC of very bright AGB stars, which on the contrary are expected to occur in a cluster with an age of about 86×10^6 yr and a turnoff mass of 5 M⊙ (Becker and Mathews 1983). It has been suggested that a rate of mass loss (either in stationary or superwind mode) much greater than customarily assumed for stars in this phase (see Iben and Renzini 1983, 1984 for all details) may result in an early termination of the AGB phase, thus accounting for the observed lack of very luminous AGB stars. As it will be discussed later on, this is not a viable explanation, or at least inadequacy of the mass loss rates is not the sole cause of disagreement.

iv) Quasi Old Clusters. The peculiar morphology of galactic as well as LMC clusters with age in the range 1 to 2×10^9 yr is another puzzling problem. In fact, Barbaro and Pigatto (1984) found that, while clusters older than than 2 to 3 x 10^9 yr generally agree with theoretical predictions for their red giant star luminosity function, theory fails in interpreting the red giant distribution in clusters of slightly lower age. In fact, their behaviour is typical of even younger clusters, in that a well developed red giant branch is not observed. On the theoretical side, this can be explained supposing that those red giants, which on the basis of the cluster turnoff mass (< 2 M⊙) are expected to be in shell H-burning phase, to develop a highly degenerate He core and therefore to eventually undergo core He flash, actually evolve as more massive stars. Since the minimum core mass for non degenerate He ignition is 0.33 M⊙, to which an initial mass of 2.2 to 2.3 M⊙ is customarily assigned, everything occurs as if stars of initial mass as low as 1.3 M⊙ or thereabouts were able to build He cores more massive (or as massive as) the above limit without passing through a phase of degeneracy in the core. The explanation of this dilemma, suggested by Barbaro and Pigatto (1984) and confirmed by Bertelli and Bressan (1985), was attributed to convective overshooting during the core H-burning phase.

v) Age Discrepancy. It has been pointed out (Hodge 1983) that ages of LMC clusters derived from the terminal AGB luminosity (Mould and Aaronson 1982 are in disagreement with ages derived from the main sequence turnoff or termination luminosity (Hodge 1983) and/or red giant clump luminosity (Flower 1984). While the ages derived from the last methods are in satisfactory agreement, they are too low when compared to Mould's and Aaronson' estimates for most of the clusters in common. Furthermore, the discrepancy is the highest for young clusters and it gets negligible for the oldest ones. To get rid of the difficulty both Mould (1983) and Hodge (1983) suggested that more mass has to be lost during the ascent of the AGB and/or the planetary nebula ejection phase. However, the same arguments against more substancial mass loss advocated for the AGB termination problem, hold even in this case and other causes of disagreement are likely to exist.

vi) AGB Star Luminosity Function. Another facet of the lack of very luminous AGB stars resides in the disagreement between theoretical and observational luminosity function for field stars of LMC studied by Reid and Mould (1984). In brief, the observed luminosity function not only shows very few stars brighter than Mb = -6, but also decreases with increasing luminosity steeper than predicted by classic models of AGB stars under any plausible assumption for the star formation rate and initial mass

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function. To overcome the difficulty Reid and Mould (1984) suggested that more mass has to be lost by AGB stars. As discussed by Renzini (1984a), Bertelli et al (1985) and Chiosi et al (1985), this is not a viable solution.

vii) Number Frequency - Period distribution of Cepheids. In a recent paper, to which the reader should refer for a better understanding of what follows, Serrano (1983) has analyzed the number frequency-period distribution of Cepheids in the light of classical models. The distribution is specified by three parameters: the short period cutoff Po, the period P1 of the maximum of the distribution and the rate at which Cepheids with P>P1 decay in number with respect to the period. These two periods, weighted on the initial mass function correspond to the minimum masses whose He-burning loops enter the instability strip at the red side and spend the whole He-burning in the loop within the instability strip. Equivalently, the period Po and P1 depend on the location and inclination in the HR diagram of the blue loop band with respect to the Cepheid instability strip. The blue loop band is known to depend critically on the chemical composition. Limiting the discussion to the sole solar vicinity to minimize effects of chemical composition, the observed difference LogP1-LogPo is in the range 0.3 to 0.6 while theory predicts 0.06 (Becker et al 1977). Furthermore, there is an excess of long period Cepheids which could be explained only by invoking a two component birth rate, more efficient for massive stars (Becker et al 1977). It is easy to show that Becker's (1981) models yield too a small LogP1-LogPo difference for any combination of chemical abundances. To overcome the problem we may either make wider the instability strip (Pel and Lub 1978) or to increase the slope of the blue loop band. This latter alternative will turn up to be the most reasonable solution. In such a case, not only the period difference is matched, but also the excess of long period Cepheids is accounted for.

2. PHYSICAL FUNDAMENTS OF EVOLUTIONARY MODELS

In this section, we briefly summarize the main ideas relative to a few points of stellar structure which are at the base of the most recent computations, even if they have not yet received general consensus.

i) Convective Overshooting

In classic stellar models, the boundary of the convective core is defined by the condition $\nabla_R = \nabla_A$, which is equivalent to say that the core may extend up to the layer where the buoyancy acceleration of convective elements vanishes. The velocity however does not get zero at this layer, implying that convective elements may penetrate (overshoot) into the formally stable radiative zones, thus increasing the mass of the convective core. Due to well known uncertainties in the physics of convection and mixing processes, contrasting conclusions have been reached by different authors, which go from considering convective overshooting negligibly small to claiming that it is of paramount importance. Among others, we recall Maeder (1975, 1976), Cogan (1975), Roxburgh (1978), Cloutman and Whitaker (1980), Maeder and Mermilliod (1981), Matraka et al (1982), Doom (1982a,b, 1985), Bertelli et al (1985a). In particular, Bressan et al (1981) have shown the importance of convective overshooting in massive stars, while Bertelli et al (1985a) have investigated its far reaching effects in the domain of intermediate mass stars. These authors describe convective overshooting by means of the mixing length theory of convection and propose a formalism containing the mixing length scale of motions ($l = \lambda$ Hp) as an adjustable parameter (λ) to be eventually fixed by comparing model results with observations. In this formalism, when $\lambda = 0$ the classic condition is recovered.

ii) Mass Loss by Stellar Wind

Luminous early type stars and late type giants and supergiants are known to lose mass from the surface at rates that may significantly affect their evolution. The subject has been reviewed so many times that a detailed presentation of current mass loss rates, rate parametrizations in terms of basic stellar quantities, and physical processes powering the wind of different types of star is superfluous. Quite an exhaustive review of those topics and appropriate referencing to original sources can be found in Chiosi and Maeder (1985), to whom we refer for more information. It suffices to recall here the mass loss rate parametrizations that have been used in model computations we are going to describe. a) Massive stars have been evolved taking into account mass loss according to the following prescription for the rates: Chiosi and Olson (1984) for O-B type stars, Barlow and Cohen (1977) for A to M supergiants, the rates however scaled to the results of Jura and Morris (1981), and Barlow et al (1981) for the so called WR stage. This is assumed to begin when the following conditions are satisfied: surface abundance of hydrogen less than 0.1 (in mass) and Teff of the model greater than 20000 °K. b) A few exploratory sequences for intermediate mass stars have been computed using either Waldron's (1984) parametrization, however rescaled as described in Bertelli et al (1985a) to obey the constraint imposed by globular clusters (Fusi Pecci and Renzini 1976), or Reimers' (1975) formula with his parameter η comprised in the range 0.3 to 1. Using Bertelli's et al (1985a) notation, Waldron's mass loss rates multiplied by the parameter $\alpha = 0.2$ to 0.3 are equivalent to Reimers' rates for $\eta = 1$.

iii) Nuclear Reaction and Neutrino Emission Rates

The major novelty in this context is the recent determination of the cross section for the $^{12}C(\alpha\ \gamma)^{16}O$ reaction by Kettner et al (1982) together with the theoretical reanalysis by Langanke and Koonin (1982). The new rate runs 3 to 5 times faster than the classic rate of Fowler et al (1975). In the usual notation this rate is parametrized by θ_α^2 . Here the new value for θ_α^2 is taken to be three times greater than the canonical value of 0.078. All other reaction rates pertinent to the evolutionary phases under consideration are as in Fowler et al (1975). Whereever appropriate, the neutrino energy losses have been included. These are from Beaudet et al (1967), using their interpolation formula which is accurate within 20%.

iv) Radiative Opacities

Opacity of the stellar material was derived by interpolation (in temperature, density and chemical composition) of the opacity tables of Cox and Stewart (1970) for most of the models computed. However, there are several independent arguments (Simon 1982; Iben and Renzini 1984; Bertelli et al 1984) suggesting that current opacity calculations may actually underestimate the true opacity in the middle temperature region (5×10^5 to 5×10^6 °K) where the main sources of opacity are the bound-bound and bound-free transitions involving occupied levels in highly ionized species of elements from carbon to iron. Starting from the modification proposed by Bertelli et al (1984) and a suggestion advanced by Renzini (1984b), Bertelli and Bressan (1985) adopted the following relation

$$\kappa = \kappa_{XY} + A\ \Delta Z \left[1 + \chi\ f(\rho,T)\ \exp(\ -\alpha\ \mathrm{Log}(T/To)^4)\right] \qquad (1)$$

where κ_{XY} is the opacity of a metal free mixture of H and He. A ΔZ gives the contribution of heavy elements to the classical opacity. This is evaluated subtracting at each value of ρ and T the opacity of a metal free mixture with given X and Y from the opacity of a mixture having the same X and Y but metal abundance Z. Furthermore, $f(\rho\ ,T)$ is a suitable function defining in the ρ-T plane a band along which the opacity enhancement given by the exponential term is allowed to occur. T_o is the central value of the temperature interval (10^6 °K). The request of matching the opacity increase proposed by Simon (1982) and Bertelli et al (1984) allows us to determine α, χ and $f(\rho\ ,T)$. Since all this is highly speculative, only a few models have been computed. They will be discussed separately.

v) The Models

Evolutionary sequences in the mass range 1 Mo to⊙ 100 M⊙ have been computed from the main sequence to the red giant tip for masses in the range 1 M⊙ to 1.6 M⊙, to the beginning of the thermally pulsing AGB phase for masses in the range 1.6 M⊙ to 6 M⊙ (Z = 0.02) or to 5 M⊙ (Z = 0.001), and beyond the C-ignition stage for larger masses. Two sets of chemical abundances have been considered, namely X = 0.700 and Z = 0.02 (Bertelli et al 1985a and 1985b) and X = 0.700 and Z = 0.001 (Angerer et al 1985). Although many sequencies have been evolved with different choices of the overshooting parameter, we will present here only the set of models for λ = 1 as according to Bertelli et al (1984,1985a) it gives the most interesting results.

3. EFFECTS OF CONVECTIVE OVERSHOOTING AND NEW θ_α^2 ON STELLAR STRUCTURE

To fully appreciate the differences with respect to standard theories of stars given by convective overshooting it is worth introducing five critical values of the initial mass which in turn define six mass ranges in which stellar evolution proceeds differently.

1) $M_i > M_{mas}$ (massive stars): above this limit stars proceed through a series of nuclear burnings in non degenerate conditions towards the construction of an iron core, subsequent photodissociation instability with core collapse and supernova explosion (see Woosley et al 1984 for all details). The current value of M_{mas} is about 12 M⊙.
2) $M_{up} < M_i < M_{mas}$ (quasi massive stars): stars in this mass range ignite carbon under mildly degenerate conditions, suffer a mild carbon flash but burn carbon non violently. Their subsequent evolution is rather complicate, but eventually terminated by core collapse leading to a supernova explosion (Nomoto 1984). These stars and those of the previous range fail to develop a highly degenerate CO core, and do not experience the thermally pulsing phase of AGB (TP-AGB). The current value of M_{up} is about 9 M⊙ for Pop I and lower than this (= 7 M⊙) for Pop II chemical composition (Becker and Iben 1979).
3) $M_{HeF} < M_i < M_{up}$ (intermediate mass stars): stars in this mass range

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ignite helium nondegenerately but following He-exhaustion develop a highly degenerate CO core. They experience a long thermally pulsing AGB phase, terminated either by envelope ejection and formation of a white dwarf (for $M_{HeF} < M_i < M_W$) or carbon ignition and deflagration in a highly degenerate core which has grown to the Chandraseckar mass of 1.4 M⊙. This event is usually referred to as a supernova of type I1/2 (Iben and Renzini 1983). The critical mass M_W above which this may occur is determined by the efficiency of mass loss by stationary wind or so-called "super wind". Current estimates set M_W around 5 M⊙ (Iben and Renzini 1983, 1984).

4) $M_i < M_{HeF}$ (low mass stars): stars in this range of mass develop a highly degenerate He core shortly after central hydrogen exhaustion. They develop the red giant branch along which they suffer significant mass loss by stellar wind. The prolonged red giant branch phase is terminated by the violent ignition of He-burning in the core (He-flash), when the core mass has grown to 0.4 to 0.5 M⊙ depending on the chemical composition. Their subsequent evolution is quite similar to that of intermediate mass stars. The classic value of M_{HeF} is 2.2 to 2.3 M⊙ for Pop I and about 1.8 M⊙ for Pop II chemical composition. Within this mass range it is worth defining another mass, M_{LC}, above which stars possess convective cores on the main sequence. This mass represents the minimum value for stars being affected by convective overshooting during their core H-burning phase. A provisional estimate sets M_{LC} at about 1.2 M⊙ for Pop I composition (Bertelli et al 1985b).

i) Core H-burning Phase

Since the major effects of overshooting on core H-burning models are already known (Maeder 1976; Maeder and Mermilliod 1981; Bressan et al 1981; Bertelli et al 1985a), the discussion will be kept very short. Models with overshooting possess more massive convective cores, run at higher luminosity and live longer than classical models. They also extend the main sequence band over a wider range of Teff. The above effects depend on λ and on the star mass. The relative increase in the mass of the convective core is $\Delta(Mr/M) = 0.16$ when λ varies from 0 to 1, while at given λ the increase in core mass by overshooting is greater at lower masses. The increase in the lifetime mimics the dependence of the increase in the core mass on the star mass. The same is true for the increase in the stellar luminosity. The variation in the range of Teff's covered by core H-burning models, namely the extension of the main sequence band, increases with the stellar mass. Massive stars ($M_i > 40$ M⊙) would spread all across the HR diagram, it were not for the contrasting effect of mass loss.

ii) Core He-burning Phase

The overluminosity caused by overshooting during the core H-burning phase still remains during the shell H and core He-burning phases. The mass of the H-exhausted core, M_{He}, and the mass of the CO rich He-burning convective core are increased by $\Delta(Mr/M) = 0.13$ when λ increases from 0 to 1. As a consequence of the higher luminosity, the lifetime of the He-burning phase (t_{He}) gets shorter in spite of the increase in the core caused by overshooting. This combined with the longer H-burning lifetime t_H makes the ratio t_{He}/t_H rather low (from 0.12 to 0.06 when the star mass varies from 1.6 M⊙ to 9 M⊙. In Table 2 we summarize the lifetimes for the set of models with X = 0.700 and Z = 0.02.

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Table 2

(Lifetimes of models with overshooting and X = 0.700 and Z = 0.02)

$M/M_\odot$	t	t(H)	t(He)	t(Heb)	t(Her)	t(He)/t(H)
1.2		4.26 (9)				
1.4		3.24 (9)				
1.5		2.73 (9)				
1.6	2.59 (9)	2.30 (9)	2.76 (8)	No loop	2.76 (8)	0.12
1.7	2.18 (9)	1.95 (9)	2.30 (8)	No loop	2.30 (8)	0.12
3.0	4.60 (8)	4.27 (8)	3.26 (7)	No loop	3.26 (7)	0.08
5.0	1.25 (8)	1.18 (8)	7.06 (6)	3.59 (6)	3.20 (6)	0.06
6.0	8.34 (7)	7.88 (7)	4.62 (6)	2.37 (6)	2.07 (6)	0.06
7.0	8.04 (7)	7.70 (7)	3.35 (6)	1.98 (6)	1.37 (6)	0.05
9.0	3.55 (7)	3.34 (7)	2.10 (6)	1.20 (6)	9.00 (5)	0.06

ages are in years; overshooting is for $\lambda = 1$; new θ^2_α

The location in the HR diagram of core He-burning models can be schematically summarized as follows. In massive stars, where mass loss may occur even during the main sequence phase, it is almost entirely dominated by this phenomenon and it may take place either partly in the red and partly in the blue, or entirely in the red, or entirely in the blue. A detailed description of this phase can be found in Chiosi and Maeder (1985). In the range of quasi massive and intermediate mass stars, it is well known that extended loops may develop. Their extension however depends on chemical composition and details of the model structure (see the discussion of Renzini 1984c on this subject), and furthermore on overshooting, θ^2_α and mass loss during the red giant phase. In brief, convective overshooting strongly decreases the loop extension, mass loss along the Hayashi line makes the loops even redder, while an increase in θ^2_α acts in the opposite sense. Finally, loop extension depends on the star mass, in general they begin and get bluer at increasing mass. Fig. 1 shows the HR diagram of intermediate mass stars evolved at constant mass whose lifetimes are given in Table 2.

iii) The Critical Masses M_{mas}, M_{up} and M_{HeF}

In virtue of the larger helium core and carbon-oxygen core left over at the end of core H and core He-burning respectively, the relation between the initial mass and M_{He} and M_{CO}, which defines the above critical masses. is different with the new models . The new critical masses are summarized in Table 3 for the two sets of chemical composition. The most important result is that both M_{up} and M_{HeF} are significantly lower in models with overshooting. This means that no AGB phase is now expected above the new M_{up}, while no prolonged RGB phase occur for stars as low as 1.6 $M_\odot$. The impact of this finding on observational front is straightforward and of paramount importance. Lower values of M_{up} and M_{HeF} have been also suggested by Renzini et al (1985), Castellani et al (1985) and Barbaro and Pigatto (1984) respectively. Remarkably, in models with overshooting, M_{HeF} has been found not to depend on Z, while its possible dependence on X has not yet been tested. Finally, we recall that a different $M(M_{He})$

Table 3

(Critical Masses M_{mas}, M_{up} and M_{HeF} : in M⊙)

X	Z	M_{mas}	M_{up}	M_{HeF}
0.700	0.020	9	6	1.6
0.700	0.001	8	5	1.6

relation now holds all over the mass range in which convective overshooting is in operation. This is particularly relevant in the domain of massive stars in conjunction with the problem of chemical enrichment per stellar generation (see Chiosi and Matteucci 1984 for a complete discussion of the topic). Furthermore, the drastic lowering of M_{up} towards M_W makes the occurrence of type I1/2 supernovae very unlikely.

iv) Effects of θ_α^2 on the chemical structure of the CO core
With the old value of θ_α^2 almost equal abundances of C^{12} and O^{16} are expected in the carbon-oxygen core, the abundance of C^{12} being moreover a decreasing function of the star's mass (Arnett 1972). With the new θ_α^2 very little carbon is left in the core, the final abundance of it being lower than 10%. This result is preliminary and has to be confirmed by more extensive calculations. Furthermore, if confirmed by other measurements of the $^{12}C(\alpha\ \gamma)^{16}O$ cross section, it may have strong implications for the subsequent evolution of intermediate and massive stars. We will touch upon this subject later on.

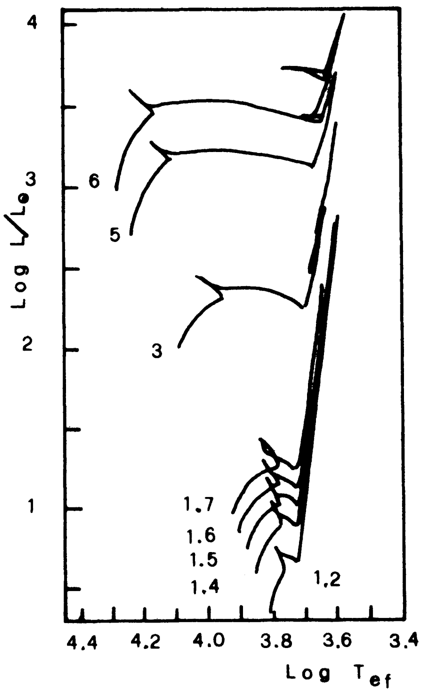

Fig. 1 The HR diagram of intermediate mass stars and low mass stars with convective overshooting (λ = 1) and new θ_α^2. The transition mass M_{HeF} is at 1.6 M⊙, while M_{up} is at 6 M⊙. The chemical composition is X = 0.700 and Z = 0.020. The models are evolved at constant mass

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4. OBSERVATIONAL IMPLICATIONS FOR MASSIVE STARS

Under the combined effects of overshooting and mass loss, the band of core H-burners may now extend up to the spectral type B1 in the luminosity range correspondent to stars of initial mass from 20 M⊙ to 60 M⊙. At higher masses the core H-burning band shrinks towards the zero age main sequence as it occurred for models with classical convective cores but losing mass at substantial rate. The advantage here is that the same result is obtained without enormous losses of mass, owing to the larger convective cores, which favour the appearence of CNO processed material at the surface. The lowering of the opacity (mostly electron scattering) will produce a smaller radius. The results for the mass range 20 M⊙ to 60 M⊙ are particularly interesting. In fact the model stars may now spend in the spectral range B0 - B1 about 20 % of their total core H-burning lifetime, which approximately amounts to three times the core He-burning lifetime. As a consequence of this, we expect in this spectral range, which approximately corresponds to the Kelvin-Helmotz gap of classical models as many stars as in all other later spectral classes. As shown by Bertelli et al (1984) this greatly alleviates the difficulty of too many stars falling outside the formal main sequence band of standard models. The core He-burning phase takes place partly in the red and partly in the blue supergiant regions for initial masses in the range 20 M⊙ to 40 M⊙ or thereabouts. Approximately 30 % of the He-burning lifetime is spent by a typical 20 M⊙ star as a red supergiant. Stars more massive than 40 M⊙, even though they may reach the red supergiant region, spend there very short time and soon run back into the blue side of the HR diagram (WR progenitors ? as suggested by Humphreys et al 1985). The behaviour of stars in the mass range 10 M⊙ to 20 M⊙ in not entirely clarified. Likely they will spend the whole core He-burning lifetime as red supergiants. 10 M⊙ to 12 M⊙ stars likely constitute the upper limit for loops to develop with Pop I composition. Table 4 shows the stellar counts by Bertelli et al (1984) for models with overshooting in the luminosity range - 7 > Mb > - 9, using the same material we presented in section I. Now 80 %

Table 4

Sp	O-B1	B2-B9	A	F	G	K	M	WR
N	432	49	10	4	1	1	29	20
N/Nt	0.79	0.09	0.02	0.01	~0	~0	0.05	0.04

of stars fall in the formal main sequence band. However, since t_{He}/t_H = 0.06, there seems to be a residual deficiency of stars amounting to about 15 %. Whether the arguments of Garmany et al (1982) and Vanbeveren (1986), in that a fraction of potential O type stars is not yet observable, or other causes of further extension of the main sequence band (up to the spectral type A0) are to be found, is not very clear. Bertelli et al (1984) analyzed the second alternative, advancing the idea that only a suitable combination of atmospheric effects caused by mass loss on the radius and a gentle increase in the middle temperature opacity could be able to reproduce the observed stellar frequencies. Their conclusion rested on and was somewhat vitiated by the assumption that stars down to

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about 20 M⊙ should have been able to generate WR stars. More recent analyses of WR star progenitors based on cluster membership (Schild and Maeder 1984 and Humphreys et al 1985) by moving upwards the progenitor mass have alleviated the discrepancy. Even if the main properties of the evolution of massive stars in occurrence of mass loss are reasonably well understood, still they are hampered by the uncertainties on mass loss rates and their parametrization. Perhaps the most interesting advancement in the theory of massive stars is related to the new cross section for $^{12}C(\alpha,\gamma)^{16}O$ reaction. In fact as a consequence of the very little carbon left in the core following He-exhaustion, massive stars may be able to skip the C-burning phase. As found by Wilson et al (1985) this has profound consequences in subsequent evolution. In fact, their 25 M⊙ star with the new rate develop a much larger iron core. The reason of it resides in the nature of carbon and neon burnings and how they affect the entropy structure of the core. In brief, with the old reaction rate C-burning ignites as a well developed, exoergic convective burning stage. While C-burning goes on, neutrino losses cool the core and cause loss of entropy. Later on a C-burning shell ignites at the border of a semidegenerate core close in mass to the Chandraseckar limit. Due to the entropy barrier set up at the border of this core, the C-burning shell cannot migrate outward. So that when oxygen and silicon burning are completed and the core eventually collapses, it does so with an iron core of about 1.33 M⊙. With the new rate, C-burning and also Ne-burning never ignite in the centre as exoergic convective burning stages. The little traces of C and Ne burn out radiatively on a very short time scale. Because there is no sufficient cooling stage, the core does not degenerate and becomes insensitive to the Chandraseckar mass. When the star reaches the collapse, it has built an iron core of about 2.2 M⊙. A black hole is likely to form in this event. It is clear that the particular value of the initial mass above which this may occur, depends on the amount of carbon present in the core which in turn depends on the star's mass.

5. OBSERVATIONAL IMPLICATIONS FOR INTERMEDIATE MASS STARS

In this section we briefly touch upon the use of models with overshooting in a few of the topics presented in section I, namely the problem of the HR diagram of intermediate age and quasi old clusters, the problem of clusters dating and age discrepancy, and finally the problem of AGB star luminosity function. Although the adopted chemical composition may not be fully suited, we shall consider only LMC clusters as they are the best laboratory where such a comparison can be successful.

i) Intermediate Age Clusters (NGC 1866)

NGC 1866 is particularly suited to this purpose, because it is well populated and because it has been studied recently by Becker and Mathews (1983) on the basis of classic models. Those authors assigned NGC 1866 a turnoff mass of about 5 M⊙, a chemical composition Y = 0.273 and Z =0.016, and an age of $86 \pm 5 \times 10^6$ yr. However they failed in reproducing the relative stellar frequencies (main sequence versus red giant stars, blue versus red giants) and the lack of bright AGB stars. Table 5 contains the star counts derived from the photometric study by Robertson (1974) plus the list of super luminous stars by Flower (1981). Since the main sequen-

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Table 5

(Stellar Counts in NGC 1866)

Note	MS	BG	RG	AGB	SLS
(1)	206 + 14	46^{+10}_{-7}	47^{+29}_{-7}	14^{+13}_{-5}	6
(2)	116	60	137	40	
(3)	21.1 (7)	0.79 (7)	0.79 (7)	<< 5 (6)	

(1) Star counts from Becker and Mathews plus the super luminous stars (SLS) of Flower; (2) Star numbers predicted by Becker and Mathews; (3) Theoretical lifetimes for a 4 M ⊙ star with convective overshooting (λ = 1) and chemical composition X = 0.700 and Z = 0.020

ce stars may be affected by incompleteness in that the majority of them fall below the survey limit (18 mag.), we prefer to use the giant star population in comparing theory with observations. Omitting all the details of the discussion which can be found in Bertelli et al (1985a) we propose the following scenario: turnoff mass of about 4 M⊙, age of about 210 x 10^6 yr, an equal percentage of blue and red giants (cfr. the lifetimes of Table 5). With the above choice, the number of main sequence stars in the observational sample would amount only to 10 % of the total pertinent to a turnoff mass of 4 M⊙. The rest falls below the survey limit. The lack (paucity in general) of very bright AGB stars can be understood by the much lower M_{up} indicated by models with overshooting. The few superluminous stars are compatible (in number) with being core C-burners (cfr. Flower et al 1980). Mass loss by stellar wind might help to remove them away from the AGB area. If all this is correct, it would imply that M_{up} for chemical composition pertinent to NGC 1866 is down to about 4 M⊙. Models of ours with X = 0.700 and Z = 0.001 yield M_{up} = 5 M⊙. Other chemical compositions have not yet been explored. Direct measurements of the chemical abundance in NGC 1866 stars would be highly useful.

ii) Quasi Old Clusters (NGC 2190 and NGC 2162)

The HR diagrams of clusters like NGC 2190 and 2162, recently obtained by Schommer et al (1984) with CCD photometry, are reflective of the lower M_{HeF} given by models with overshooting, since these two clusters turn up to be similar to those studied by Barbaro and Pigatto (1984). In fact, in the study of Chiosi and Pigatto (1985) an age of about 1 x 10^9 yr has been assigned and more important it has been shown that they possess a single peak luminosity function for red giants. The correspondent mass is about 1.8 M⊙. This well to models with overshooting which predict M_{HeF} as low as 1.6 M⊙ and core He-burning to occur along the Hayashi line, to which a clump of red giants corresponds on observational side. With classical models of the same age and chemical composition, there would be a prolonged red giant branch followed by core He-burning in the horizontal branch (red in this case). A double peak luminosity function for red star is therefore expected, contrary to what observed.

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iii) Clusters Dating and Age Discrepancy

As already anticipated in section I, three methods exist to date clusters, namely the luminosity of the main sequence turnoff (and/or termination), the luminosity of the He-burning red giants, the luminosity of the bright AGB stars. Each of those rests upon suitable luminosity versus age relationships derived from theoretical models. Unfortunately, among other factors, the theoretical relations depend on the chemical composition, which has to be specified a priori. Good abundance determinations are available only for very few clusters. Despite of it, age compilations have been derived (Hodge 1983; Flower 1984; Mould and Aaronson 1982). The results are very discouraging as ages derived by different methods in general disagree by large factors. It goes without saying that each method competes with intrinsic observational as well as theoretical difficulties that often invalidate the whole results.

a) The turnoff method is hampered by the lack of very good photometric data down to magnitudes faint enough to delineate the unevolved portion of the main sequence. This is particularly severe for LMC clusters older than a few 10^8 yr or thereabouts.

b) The red giant luminosity method requires that the luminosity of core He-burners be a monotonic function of the age. If this is true (the luminosity decreases with increasing age) for stars more massive than M_{HeF}, it does no longer hold below this limit, firstly because the relation flattens out, secondly because the luminosity of core He-burners generated by stars less massive than M_{HeF} is higher than that of stars in the range $M_{HeF} < M < M_{HeF} + 0.5$ Mo (Renzini and Buzzoni 1986). The relation is somewhat bivariate at M_{HeF}. Therefore, before using the red giant method, care has to be payed to ascertain the type of cluster we are dealing with.

In Fig 2 we present the above calibrating relationship derived from models with overshooting and two values of metallicity.

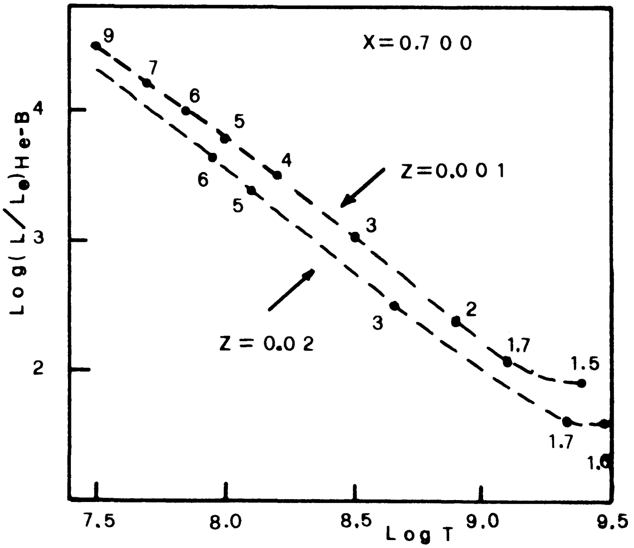

Fig. 2 Luminosity of core He-burning models versus age relationships. The evolutionary models allow for convective overshooting as described in the text. Along each curve the stellar masses are indicated. The chemical compositions are indicated in the panel

c) The AGB luminosity method can be applied only to clusters with turnoff mass below M_{up}. However the greatest uncertainty with this method resides in the low number of stars in AGB which makes the identification of the maximum AGB luminosity quite uncertain.
Chiosi et al (1985) revisited the whole problem of clusters dating in the light of models with overshooting. The results can be summarized as follows:
1) Due to the higher luminosity and longer lifetime of core H-burning of the new models, we expect that for any given turnoff (or termination) magnitude of clusters whose turnoff mass has a convective core, the correspondent age is greater than classic estimates. Similar increase in the age is also expected from the red giant method for all clusters which have a turnoff mass greater than M_{HeF}. The relations below may be used to convert ages derived from classical models into the new ones,

$$\text{Log } t = 0.78 \text{ Log } t_{old} + 2.337 \qquad \text{(turnoff method)}$$

$$\text{Log } t = 1.05 \text{ Log } t_{old} + 0.092 \qquad \text{(red giant method)}$$

where ages are given in years.
2) Following the procedure outlined by Iben and Renzini (1983, 1984) however adapted to the new models, a novel relation between maximum AGB luminosity and age is derived. The rate of mass loss during the red giant and AGB phases is from Reimers (1975) with $\eta = 1$ or equivalently from Waldron (1984) with $\alpha = 0.2$ (cfr. section II). Ages derived from the novel relation are only modestly changed by overshooting. This surprising result can be understood as due to the fact that AGB evolution is mainly driven by the CO core mass and it depends little on the past history. What is actually changed is the correspondence between initial mass and total lifetime.
The most important result is that ages based on the main sequence turnoff get closer to those from AGB luminosity. Even with all reservations caused by the uncertainties discussed insofar, the so-called Age Discrepancy if not completely ruled out is greatly alleviated when overshooting is taken into account.

iv) The AGB Luminosity Function For Field Stars In LMC

Here we discuss the AGB star luminosity function obtained by Reid and Mould (1984) for a selected area of LMC. The luminosity function (number of stars per magnitude bin) is shown in Fig 3 and compared to the prediction based on the standard theory of AGB stars. All details relative to the procedure and assumptions made concerning the star formation rate and initial mass function are given in Reid and Mould (1984) and therefore omitted here. It suffices to recall that the particular case shown in Fig 3 is derived assuming a constant star formation rate and the Salpeter mass function. Chiosi et al (1985) have repeated the whole analysis using models with convective overshooting in the previous evolutionary phases. The mass loss rates were the same as reported in the previous paragraph. All other assumptions are as in Reid and Mould (1984). Nevertheless, two major changes are important. First, the contribution of early AGB stars neglected by Reid and Mould (1984) was taken into account. Second, it has been found that also normal red giants (core He-burners) with initial mass

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in the range 4.5 M⊙ to 7 M⊙, being overluminous because of overshooting, may contaminate the lower magnitude bins. No such contamination was expected with classical models because the stars in question were either too faint or too short lived to appreciably alter the statistics. After

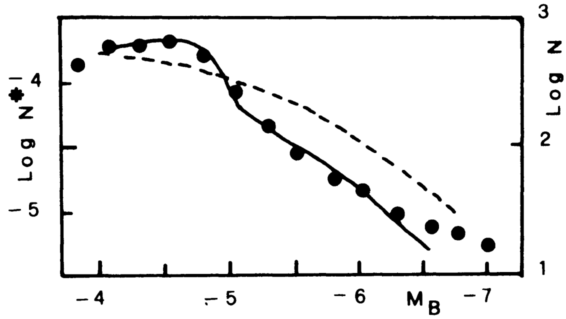

Fig. 3 The luminosity function for AGB stars. N and N* are the observed and predicted number of stars per magnitude bin. They differ by a normalization factor. The dashed line visualizes the standard case by Reid and Mould, while the continuous line shows the result for models with overshooting (see text)

normalization of the theoretical stellar numbers per magnitude bin at the second 0.25 mag. bin, we find the relation shown in Fig 3. The agreement with observation is remarkably good. The net improvement is mostly due to the novel features of models with overshooting, and only partly to the slight increase in the mass loss rate we have adopted. The same result would not have been possible by varying the mass loss rate alone.

6. MODELS WITH VARIED OPACITY

In addition to the calculations presented by Bertelli et al (1984) with opacity given by eq. (1), a few exploratory sequences have been computed by Bertelli and Bressan (1986) in the domain of intermediate mass stars. The results are as follows:

i) In absence of convective overshooting the models perform very extended loops in the HR diagram, as already shown by Robertson (1972), who first proposed such a modification of standard radiative opacities.

ii) In models with overshooting the blue loops are strongly suppressed for all masses greater than 5 M⊙ when ΔZ in eq. (1) is 0.02. This almost independently of λ . On the contrary, when $\Delta Z = 0.01$, loops though reduced are still there. They tend however to get narrower at increasing mass.

iii) Those numerical experiments bring an interesting feature to the morphology of blue loop band of intermediate mass stars, namely a C-shaped behaviour whose blue apex is much more sensitive to metallicity than with standard opacity. This may have important observational implications. In particular, the Cepheid instability strip could be crossed two times by the blue loop band on quite long time scales or even be overlapped by this under favourable circumstances (efficient overshooting and/or high metal content). It is easy to foresee how the number frequency versus period relation (cfr. section I) would change in this case. Amazingly enough, the bimodal distribution of SMC Cepheids (Becker et al 1977) could be accounted for in a simple way. This is however highly speculative and more work has to be done to cast light on the need (if any) of opacities different from those currently in use.

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7. CONCLUDING REMARKS

Before concluding this paper, we like to touch upon two more points that may be affected by models with overshooting.

i) The Number Frequency - Period Distribution Of Cepheids

In section I, we reported on the difficulty encountered by classic models for core He-burning stars in reproducing the observed number frequency - period distribution of Cepheids, and the need of a steeper band of core He-burning stars in the HR diagram. This problem has been addressed by Bertelli et al (1985a), who showed how models with overshooting possess the required feature and can almost match the observed period difference LogP1-LogPo.

ii) Integrated Colour Versus Age Relation For Star Clusters

With the aid of the new models, Bertelli et al (1986) have calculated the integrated colours - (B-V)o and (U-B)o - versus age relationship for clusters having turnoff mass down to 1.6 M⊙. All phases, from the zero age main sequence to either the termination of the AGB (for stars with initial mass in the range 1.6 M⊙ to 6 M⊙) or beyond the core He-exhaustion stage (for stars more massive than 6 M⊙), have been included. Starting from isochrones, stars are distributed along a given isochrone by means of a "Montecarlo" technique weighted on the initial mass function. A huge number of stars per model cluster has been considered in order to simulate and approach the analytical case. The resulting colour versus age relationships seem to better fit the colours of a few clusters of LMC for which the age was independently derived from the turnoff magnitude (Chiosi et al 1985). Particularly interesting is the abrupt change in the slope of the (B-V)o versus age relation occurring at about 1×10^9 yr, which could be related to the discontinuity seen in the (B-V)o versus cluster type classification parameter of Searle et al (1980). See also the thorough discussion on this topic by Renzini and Buzzoni (1986). More work is under way to cast light on this problem.

In this paper we reported on studies of the effects of convective overshooting on stars of all masses and evolutionary phases in which this phenomenon may be effective. The aim was not only to discuss convective overshooting from the viewpoint of general interest toward this particular physical process, but also to find astrophysical tests of its occurrence in real stars and hopefully to indirectly assess the actual extension of convective cores. Looking at the results we have presented insofar, convective overshooting turns up to be a very promising tool for removing or at least alleviating some of the discrepancies that were known to exist between current theories of stellar structure and crucial observational facts. Whether or not the arguments presented in this paper have been convincing and the goal achieved is difficult to say. Certainly this line of work deserves more careful studies.

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