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Some analytical solutions for problems involving highly frictional granular materials

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Granular materials are extensively used throughout many industries. In countries like Australia and South Africa, national economies are critically dependent on the agricultural and mining industries, and it is important to understand bulk material behaviour in order to handle these materials properly. There are many diverse approaches to modelling granular materials. In this thesis, the Coulomb-Mohr continuum mechanical theory of granular materials is utilised, and some exact analytical solutions are presented for both stress and velocity fields. These solutions are formally valid provided $\sin\phi = 1$, where ϕ is the angle of internal friction, and they provide limiting solutions for so called highly frictional granular materials, for which $\sin\phi \approx 1$. The solutions so obtained are exploited for three real industrial granular problems, namely gravity flow from converging hoppers, the stress distribution in stockpiles and finally, the stress distribution within rat-holes.

The flow of granular materials, in the presence of gravity, through a converging hopper is an important problem that occurs in many industrial processes. Here, for highly frictional granular materials for both wedge and conical hopper flows, exact analytical solutions are obtained to determine the stress and velocity fields within the neighborhood of the hopper outlet. For the same problem, these exact solutions are exploited as the leading order terms in a regular perturbation series, where the correction terms are of order $1 - \sin\phi$. In this way, approximate analytical solutions are obtained that can be used to describe the behaviour of more realistic granular materials.

The practice of storing granular materials in stockpiles occurs throughout the world in many industrial situations. In this thesis, exact solutions for highly frictional granular materials are used for the determination of the stress distributions within two-dimensional and axially symmetric stockpiles. Stockpile profiles considered here include the standard wedge and cone, as well as two-dimensional parabolic and axially symmetric cubic curved shaped stockpiles. We emphasise that curved stockpiles have not previously been considered in the literature. Our model assumes that the stockpile is composed of two regions,

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namely an inner rigid region and an outer yield region. The exact solution applies in the outer yield region, which is then extended continuously into the inner rigid region.

A rat-hole is the general term used to describe those stable cavities, which frequently occur in storage hoppers and stockpiles, whose formation prevents further material falling through the outlet. In this thesis, we present exact solutions for both two-dimensional and axially symmetric rat-holes, where it is assumed that the highly frictional granular material has flowed through a central outlet, until the point is reached when there is no further flow of material. The remaining material is assumed to be in limiting equilibrium, that is static but on the point of yield. In both two dimensions and axial symmetry, two types of rat-holes are studied. The first type has a sloping upper surface of infinite extent, where the material is resting on a sloping rigid base containing an infinitesimal central outlet. The second type has a curved upper surface, where the material is resting between vertical walls and a horizontal rigid plane containing an infinitesimal central outlet. Again, we emphasise that neither of these rat-hole problems have previously been investigated in the literature.

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