DETERMINATION OF THE DIVISION CORRECTIONS OF THE BORDEAUX DECLINATION CIRCLE BY THE BENEVIDES-BOCZKO METHOD

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Abstract. - The automatic declination reading system implemented on the Bordeaux meridian circle with a new divided circle is shortly presented. The determination of the division errors by the Benevides-Boczko method was carried out in December 1982 and in March 1983 : the standard deviation between the two sets of corrections is about 0.015".

1. INTRODUCTION

A major source of errors in the declination determination with a meridian circle arises from the measurement of the instrument setting angle. At the present time the precision of the absolute angular encoders is at best about 0.20" on the whole circle (360°), but the precision required to match with that of the focal photoelectric micrometers is ten times better, i.e. 0.02".

The only efficient solution implemented successfully up to now on all old and modern meridian circles is a glass or metallic graduated circle with 2 N theoretically equidistant lines, the interpolation between these lines being carried out by visual or photoelectric microscopes.

The declination circle of the Bordeaux automatic meridian instrument is a divided steel circle with 7200 lines (1 line each 3') engraved on a nickel track of 500 mm diameter. The accuracy claimed by the manufactory (Société Genevoise d'Instruments de Physique) is ± 1" for each line.

The interpolation is obtained from 4 photoelectric scanning microscopes (Fig. 1). In the focal plane of each microscope is a narrow slit (16 μ × 0.8 mm) lighted by optical fiber and mounted on a small moving carriage. The carriage is driven by a linear motor, which is a modified loudspeaker coil, and its displacement on a range of ± 1 mm is measured by a linear displacement transducer (TESA differential transformer) with improved linearity. The image of the slit is projected on the track of the divided circle and reflected towards a photomultiplier by a beam-splitter cube. During the scanning, the output signal of the photomulti-

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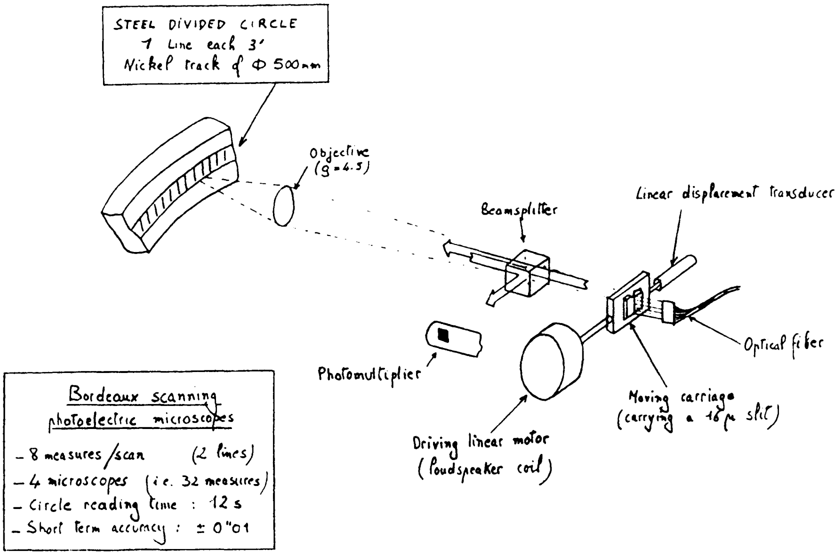

Figure 1. Schematic view of a photoelectric microscope.

plier is modulated by the two line profiles with a contrast of about 65-70 percent (Fig. 2). When the amplitude of this signal is equal to a first stable reference level, the scanning is stopped, and after a short time of stabilization, the carriage position is measured. This procedure is repeated four times for each line with two reference levels, a complete declination circle reading consists thus of 8 × 4 measurements obtained in approximately 12 seconds.

The short-term precision of a setting angle measurement with four microscopes is about 0.01" during 15-20 minutes. As a matter of fact, this number is severly affected by the variations of outdoor temperature during the night.

In order to eliminate periodic errors, mainly due to mechanical imperfections of the alignment between the circle and microscopes (eccentricity of the circle, errors of perpendicularity and also pivot irregularities), the microscopes are mounted by pairs (M_1, M_3) and (M_2, M_4) : each pair reads (as usual) diametrically opposite divisions (Fig. 3). The (M_1, M_3) pair is fixed, the second pair (M_2, M_4) can be rotated around the pillar head axis and clamped in a selected position. The angle between (M_1, M_3) and (M_2, M_4) can be varied within the range 30°- 90°. During the routine observations, this angle is kept fixed at 90°.

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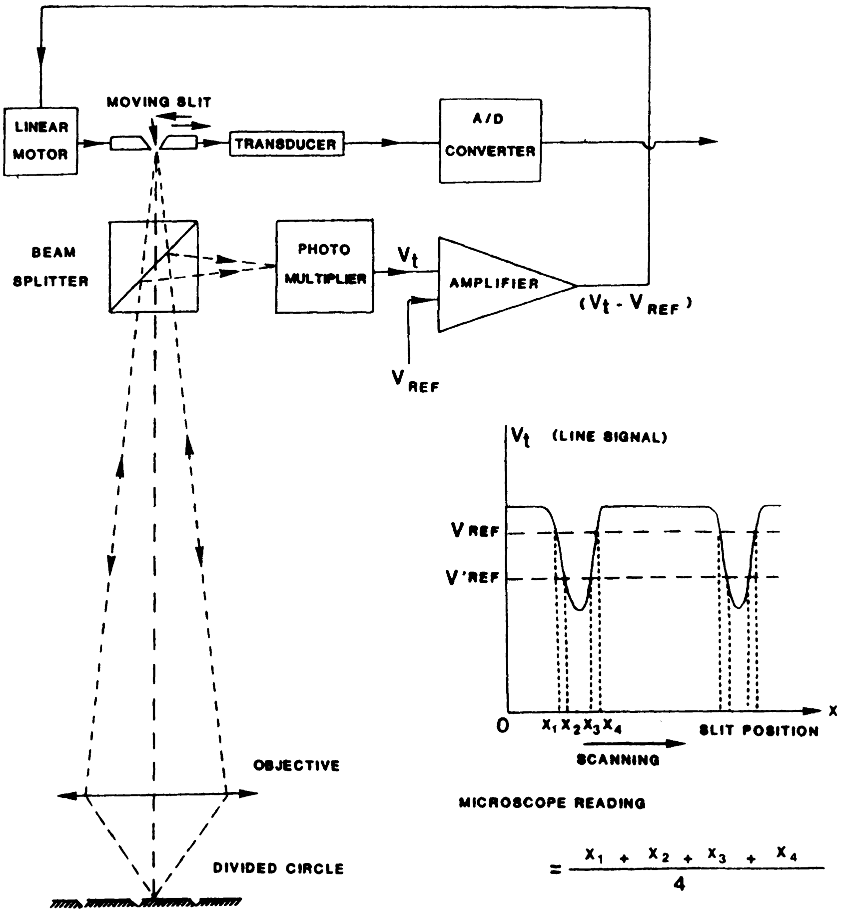

Figure 2. Working diagram of the microscopes.

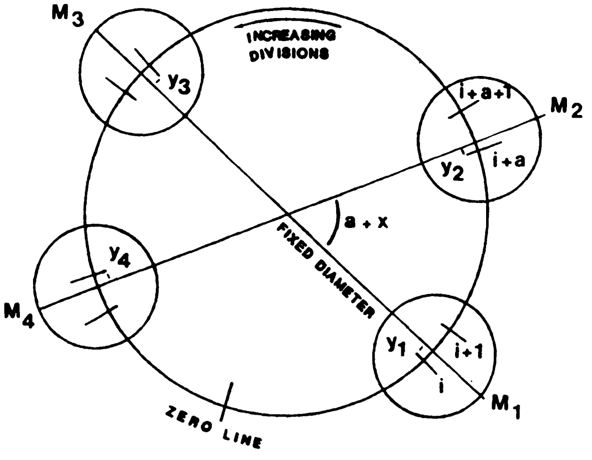

Figure 3. Principle of the measurement method.

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2. MEASUREMENT METHOD : "ROSETTE"

In order to establish the table of the N diameter corrections (N = 3600), we have at our disposal the angle between the two pairs of microscopes. This aperture angle is set at a value corresponding approximately to $(n \times 3')$, i.e. $(n \times 3') + x = a + x$. The stability of x being about ± 0.01" during 15 minutes, we can consider that during that time interval the angle $a + x$ is a convenient angular reference.

The angular distance between the diameter i and the diameter "zero" will be $(i \times 3') + C_i$ where C_i is the diameter correction to be determined. From Fig. 3, it can easily found that

$$C_i - C_{i+a} + x = \frac{y_1 + y_2}{2} - \frac{y_1 + y_3}{2} = r_i$$

with C_i correction for the diameter i,
C_{i+a} correction for the diameter $i + a$,
x unknown error on the aperture angle a,
y_2, y_4 individual interpolation measurements on the microscope pair (M_2, M_4),
y_1, y_3 individual measurements on (M_1, M_3).

If we can suppose that x is constant during groups of up to 25 complete measurements (including automatic instrument setting and complete microscope readings), we shall be able obtain, using several apertures a, a sufficient number of condition equations for a good global determination of the 3600 diameter corrections by the least-squares method.

Measurements are therefore organized in groups so-called "rosettes" at within each of which x is assumed to be constant. Let us consider a measurement series obtained through successively turning the instrument in equal steps (approximately the aperture angle a). At each step the diameter which was observed through the pair of microscopes (M_1, M_3) is thus brought into the field of the other pair (M_2, M_4). If after p such measurements we arrive at the starting diameter, the measurement closed series is called a "rosette of aperture a", and we have : $pa = Nu$, with u being an integer (i.e. a "rosette" scans an integral number of times 180°). The number of independent "rosettes of aperture a" is : $s = N/p$ and for each "rosette" starting on diameter j we have :

$$x_j = \sum_{i=1}^{p} r_{ij} / p$$

That relation means only that the angle between the microscope pairs can be directly obtained from the closing error of the corresponding rosette.

3. SOLUTION BY THE BENEVIDES-BOCZKO METHOD

Let us consider the case of three apertures a, a' and a", each of which will actually be used for our system. For each aperture, a conve-

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nient rosette period is chosen : p, p', p" corresponding to the number of independent rosettes : s, s', s". The measurements will yield 3 N independent equations of condition with $N + s + s' + s''$ unknowns, where N is the number of diameter corrections, and $s + s' + s''$ the total number of rosettes equaling the total number of angle errors x between the microscope pairs.

Following the notation of Benevides and Boczko (1981), we obtain three sets of condition equations corresponding to the three aperture angles :

$$C_{j+(k-1)a} - C_{j+ka} + x_j = r_{j+(k-1)a}$$

(with $j = 1, 2, \ldots s$ and $k = 1, 2, \ldots p$, indices greater than N being taken modulo N)

$$C_{j+(k-1)a'} - C_{j+ka'} + x'_{j+(k-1)a'}$$

$$C_{j+(k-1)a''} - C_{j+ka''} + x''_{j+(k-1)a''}$$

If p, p', p" are chosen so that their minimum common multiple is N and if we impose the auxiliary condition $\sum_{1}^{N} C_h = 0$, then the minimum length solution of the system of normal equations is easily obtained from the formulae

$$b_j = (r_j - r_{j-a}) + (r'_j - r'_{j-a'}) + (r''_j - r''_{j-a''})$$

$$m_{hk} = \frac{1}{N} \sum_{\ell=1}^{N-1} d_\ell^{-1} \cos \frac{2\pi\ell}{N} (h - k)$$

with $d_\ell = 6 - 2\cos\left(\frac{2\pi\ell a}{N}\right) - 2\cos\left(\frac{2\pi\ell a'}{N}\right) - 2\cos\left(\frac{2\pi\ell a''}{N}\right)$ being the eigen-values of the symmetric matrix of the normal equations

and $$\boxed{C_h = \sum_{k=1}^{N} m_{hk}\, b_k}$$

The generalization to any number a of apertures is obvious from these formulae.

4. RESULTS

A first test of the method was carried out with 3 aperture angles for only 60 diameters (1 diameter each 3 degrees). The classical method of Bruns-Zverev (1954), needing 5 apertures, was also applied. The standard deviation between the C_k obtained by the two methods was 0.10" after centering of the zero diameter correction.

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Two complete determinations were carried out

a) on December 1982 with 3 angles : 33°45', 70°00' and 57°36' (p = 16, p' = 18, p'' = 25) ;

b) on March 1983 with 4 angles : 33°45', 70°00', 57°36' and 48°00' (p = 16, p' = 18, p'' = 25, p''' = 15).

For each aperture, the measurement series scanning 3600 diameters was achieved in 10-12 hours.

For each determination of the 3600 C_k, we have derived 1800 corrections $E_k = \frac{C_k + C_{k+1800}}{2}$ to be applied to the normal setting angle measurement for each star observation (since in that case the mean measurement of two perpendicular diameters is used).

The maximum difference found between the two sets of corresponding E_k was 0.030". For 50 corrections E_k only, this difference is in the range 0.020" to 0.030".

In addition to the fact that a minimum number of apertures is used, it is a great advantage of the Benevides-Boczko method that it makes it possible to obtain easily the covariance matrix. Since all the divisions are measured an equal number of times, the final variance of the C_k diameter corrections is constant over the whole circle.

If μ is the error on one microscope reading, the relative variance is defined as $V = \varepsilon^2/\mu^2$, with $\varepsilon^2 = \mu^2(1/N) \sum_{k=1}^{N-1} d_k^{-1}$.

If the total number of microscope readings is 4 mN (with m = number of apertures), the efficiency of the method is

$$R = (N - 1) / 4\,mNV$$

The choice of the apertures was determined by the following conditions :

- p, p' and p'' have N as a minimum common multiple,
- the least eigenvalue d_k of the matrix of the normal equations has a maximum value. Then the variance V is minimum and the efficiency is maximum.
- p, p' and p'' are smaller than 25 (corresponding to the required stability of the aperture during one rosette),
- a, a' and a'' are greater than 30 degrees, limit which is imposed by the physical mounting of the microscopes.

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Number of readings 4 mN	Relative variance V	Efficiency R	Microscope fixations
10800 × 4	0.241	0.35	3
14400 × 4	0.152	0.41	4

The rough values of the 3600 diameter corrections C_k are given on Fig. 4. The maximum amplitude of the first order Fourier term (period 180°) is about ± 0.50", due probably to residual errors in the mounting of the microscopes and the circle. Furthermore the maximum value of the corrections E_k used in the star observations is 0.49".

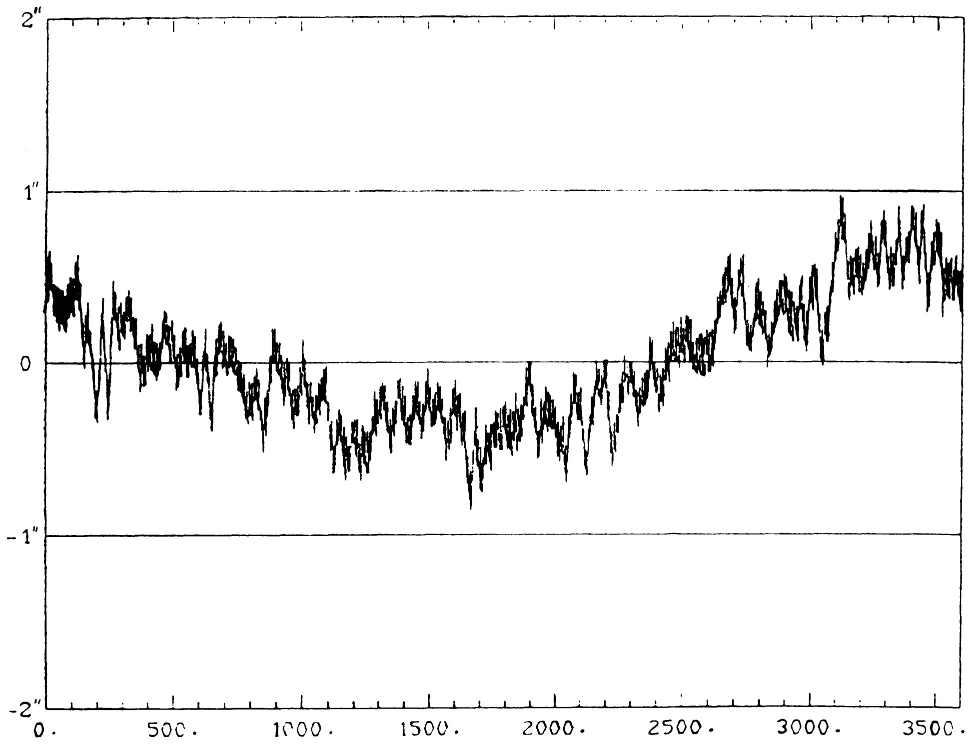

Figure 4. Individual values C_k of the diameter corrections.

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Discussion:

van ALTENA: Do you expect that these division errors will change with time, that there will be a settling of the system?
REQUIEME: We have right now no knowledge of the long-term behavior of our divided circle, especially with the variations of the temperatures. We intend to repeat the complete determination of our 3600 corrections every two years or more often.

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