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Semigroups of differentiable functions

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For over thirty years it has been known that the semigroup, under composition, of all mappings of an arbitrary set into itself has the property that every automorphism is inner. K.D. Magill, Jr, in the past ten years has shown that this property is held by many semigroups of functions and relations, in particular by the semigroup of all differentiable functions from the reals into the reals. The only new result given in the first chapter shows that the semigroup of Borel measurable functions on any T_1 topological space has the Magill property, namely the property that every automorphism is inner.

Even more recently Sadayuki Yamamuro has written a number of papers directed towards generalising the above result of Magill to semigroups of differentiable mappings defined on certain classes of locally convex spaces. The object of this thesis has been to continue that study. That the semigroup of Fréchet differentiable functions on an *FM*-space has the Magill property is the essential content of the second chapter.

Showing that a semigroup of differentiable functions has the Magill property is closely related to showing that the algebraic structure of the semigroup characterises both topologically and algebraically the locally convex space on which the functions are defined. To show that the Magill property holds for subsemigroups of those considered above then becomes of interest. For this reason semigroups of many times Fréchet differentiable mappings on *FM*-spaces are considered in the third chapter, and it is shown that they too possess the Magill property, by using the results of Chapter 2 as the first step in an inductive argument. Many times continuously Fréchet differentiable functions are likewise treated.

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An alternative proof of the result for many times continuously differentiable maps on a finite dimensional Banach space is given in Chapter 4. In this case the problem is equivalent to showing the differentiability with respect to the parameter of a one-parameter group of differentiable mappings, and so the classical theorem of Bochner and Montgomery may be applied. Further attention is also given in this chapter to the characterisation problem mentioned above. Using the notion of S-category due to Bonic and Frampton we are able to give two theorems in this direction. Under certain conditions it is also shown in Chapter 4 that if every automorphism of the group of invertible elements (units) in a semigroup is inner, then the same property will hold for the semigroup.

Admissibility of a class of spaces, a concept introduced by Magill, is extended in the final chapter to provide a framework in which to view the results. G.W. Mackey has shown that the group of continuous, linear, invertible mappings on a Hilbert space does characterise the space, but we are able to show that there exists an automorphism of this group which is not inner. The main theorem of the chapter then shows that for a large number of semigroups which contain this group, automorphisms which fix the group are inner. Certain 'd-automorphisms' of semigroups of differentiable functions are then shown to be inner.