SOURAV TARAFDER, ***Non-Classical Set Theories and Logics Associated With Them***, University of Calcutta, India, 2017. Supervised by Mihir Kr. Chakraborty and Benedikt Löwe. MSC: 03E70, 03B53, 03E40, 03B50. Keywords: paraconsistent logic, set theory, algebra-valued models, ordinal numbers.

Abstract

The theory of *algebra-valued models of set theory* was initiated in the 1960s by Dana Scott, Robert M. Solovay, and Petr Vopěnka. They took a model of set theory V and a Boolean algebra $\mathbb{B}$ to construct a new algebra-valued model of set theory $V^{\mathbb{B}}$. If the algebra is a Boolean algebra, this model will be a model of classical set theory ZFC.

If the algebra used is not a Boolean algebra, then the resulting model can be a model of nonclassical set theory. This was first done by [1] with Heyting algebras to construct models of intuitionistic set theory and later by Takeuti, Titani, Kozawa, and Ozawa for various lattices to obtain models of quantum set theories

In this thesis, we generalise this approach to *deductive reasonable implication algebras* and show that $V^{\mathbb{A}}$ becomes an algebra-valued model of the some or all of the axioms of the *negation-free fragment* of ZFC (cf. also [2]).

We also study a particular example of such an algebra, the three-valued matrix PS_3 which gives a semantics of the paraconsistent logic $\mathbb{L}\mathrm{PS}_3$ (i.e., $\mathbb{L}\mathrm{PS}_3$ is sound and complete with respect to PS_3), and show that $\mathbb{L}\mathrm{PS}_3$ is a maximal paraconsistent logic relative to classical logic (cf. also [4]).

Combining these two results, we obtain an algebra-valued model V^{PS_3} which is a model of paraconsistent set theory considerably different from other paraconsistent set theories that have been proposed. In particular the axiom scheme of comprehension remains invalid in this model.

We study the properties of the set theory validated in V^{PS_3}. Its paraconsistency is closely related to the fact that the set theory violates Leibniz's law of indiscernibility of identicals, i.e., being equal does not enforce that all properties are shared. We study the representation of natural numbers and ordinal numbers and prove that analogues of mathematical induction and Cantor's theorem are valid in V^{PS_3} (cf. also [3]).

REFERENCES

[1] R. J. GRAYSON, *Heyting-valued models for intuitionistic set theory*, ***Applications of Sheaves, Proceedings of the Research Symposium on Applications of Sheaf Theory to Logic, Algebra and Analysis held at the University of Durham, Durham, July 9–21, 1977*** (M. P. Fourman, C. J. Mulvey, and D. S. Scott, editors), Lecture Notes in Mathematics, vol. 753, Springer, Berlin, 1979, pp. 402–414.

[2] B. LÖWE and S. TARAFDER, *Generalized algebra-valued models of set theory*. ***Review of Symbolic Logic***, vol. 8 (2015), no. 1, pp. 192–205.

[3] S. TARAFDER, *Ordinals in an algebra-valued model of a paraconsistent set theory*, ***Logic and Its Applications, 6th International Conference, ICLA 2015, Mumbai, India, January 8–10, 2015*** (M. Banerjee and S. Krishna, editors), Lecture Notes in Computer Science, vol. 8923, Springer-Verlag, Berlin, 2015, pp. 195–206.

[4] S. TARAFDER and M. K. CHAKRABORTY, *A paraconsistent logic obtained from an algebra-valued model of set theory*, ***New Directions in Paraconsistent Logic, 5th WCP, Kolkata, India, February 2014*** (J. Y. Beziau, M. K. Chakraborty, and S. Dutta, editors), Proceedings in Mathematics & Statistics, vol. 152, Springer, New Delhi, 2016, to appear.

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HUGO NOBREGA, ***Games for Functions*: *Baire Classes, Weihrauch Degrees, Transfinite Computations, and Ranks***, Universiteit van Amsterdam, The Netherlands, 2018. Supervised by Benedikt Löwe and Arno Pauly. MSC: 03E15, 03D30, 03D60, 91A44. Keywords:

descriptive set theory, computable analysis, game characterizations, function classes, generalized computable theory.

Abstract

Game characterizations of classes of functions in descriptive set theory have their origins in the seminal work of Wadge, with further developments by Van Wesep, Andretta, Duparc, Motto Ros, and Semmes, among others. In this thesis we study such characterizations from several perspectives.

We define modifications of the game characterization of the Borel functions by Semmes [3], obtaining game characterizations of the Baire class α functions for each fixed $\alpha < \omega_1$. Some of our results were independently proved by Louveau and Semmes in unpublished work. We also define a construction of games which transforms a game characterizing a class Λ of functions into a game characterizing the class of functions which are piecewise Λ on a countable partition of their domains by $\mathbf{\Pi}^0_\alpha$ sets, for each $0 < \alpha < \omega_1$.

We then define a framework of parametrized Wadge games by using tools from computable analysis, and show how the choice of parameters can be used to fine-tune what class of functions is characterized by the resulting game. As an application, we recast our games characterizing the Baire classes into this framework.

Furthermore, we generalize our game characterizations of function classes to generalized Baire spaces, i.e., the spaces of functions from an uncountable cardinal to itself. We also show how the notion of computability on Baire space can be generalized to the setting of generalized Baire spaces, and show that this is indeed appropriate for developing a generalized version of computable analysis by defining a representation of Galeotti's generalized real line (cf. [1]) and analyzing the Weihrauch degree of the intermediate value theorem for that space (cf. also [2]).

In the final part of the thesis, we show how the game characterizations of function classes discussed lead in a natural way to a stratification of each class into a hierarchy, intuitively measuring the complexity of functions in that class. This idea and the results presented open new paths for further research.

REFERENCES

[1] L. Galeotti, *A candidate for the generalised real line*, ***Pursuit of the Universal, 12th Conference on Computability in Europe, CiE 2016, Paris, France, June 27–July 1, 2016, Proceedings*** (A. Beckmann, L. Bienvenu, and N. Jonoska, editors), Lecture Notes in Computer Science, vol. 9709, Springer, Cham, Switzerland, 2016, pp. 271–281.

[2] L. Galeotti and H. Nobrega, *Towards computable analysis on the generalised real line*, ***Unveiling Dynamics and Complexity, 13th Conference on Computability in Europe, CiE 2017, Turku, Finland, June 12–16, 2017, Proceedings*** (J. Kari, F. Manea, and I. Petre, editors), Lecture Notes in Computer Science, vol. 10307, Springer, Cham, Switzerland, 2017, pp. 246–257.

[3] B. Semmes, ***A game for the Borel functions***, Ph.D. thesis, Universiteit van Amsterdam, 2009.

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Rose Weisshaar, ***Some Results in Computability Theory***, University of Notre Dame, USA, 2019. Supervised by Julia Knight. MSC: 03D99. Keywords: mathematical logic, computability theory.

Abstract

We consider the question of *universality* among computable ω-branching trees. In particular, we construct a computable ω-branching tree T_{KP} whose paths compute the complete