PROPAGATION OF LARGE AMPLITUDE PULSAR WAVES

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Large amplitude waves (hereafter l.a.w.) have been studied mainly in connection with pulsars. A rotating neutron star, with an intense non-aligned magnetic dipole field, surrounded by a vacuum, radiates beyond the light cylinder distance, large amplitude electromagnetic vacuum waves of very low frequency (Ostriker and Gunn, 1969). In such a rotating configuration electromagnetic effects completely dominate and imply the existence of a relativistic plasma in the pulsar magnetosphere (Goldreich and Julian, 1969). Because the oblique vacuum model predicts a value of 3 for the braking index of the pulsar whereas the observed or computed values are different, a realistic model has to include both the relativistic plasma outflow and the electromagnetic wave emission. The vacuum wave model has been changed to include self-consistent plasma effects (Asséo et al., 1975; Asséo et al., 1978) in plane geometry and inhomogeneities linked to spherical geometry (Asséo et al., 1981). This results in very restrictive conditions for the possibility of propagation of the l.a.w.

The strength of the wave ν_1 is defined by an invariant Lorentz parameter: the ratio of the cyclotron frequency of a particle in the magnetic field of the wave to the frequency ω of the wave itself; for pulsars $\nu_1 \leqq 10^{11}$. In such intense waves, electrons and ions behave similar: their inertia is dominated by the kinetic energy they acquire in the wave and not by their rest mass. We thus assume that the wave propagates in an electron-positron plasma. Such plasmas may exist in the pulsar magnetosphere as shown e.g. by the pulsar polar cap models (Sturrock, 1971; Ruderman and Sutherland, 1975; Cheng and Ruderman, 1977; Arons and Scharlemann, 1979). We restrict ourselves to superluminous, linearly polarized electromagnetic waves, depending on the phase variable only, since these, and only these, may reach arbitrarily large amplitudes and represent the strong radiated wave involved in pulsar theory (Kennel and Pellat, 1978). In the plane homogeneous case this wave has a characteristic sawtooth shape. Its phase velocity βc depends on its amplitude. Its dispersion relation is characterized by the particle flux transported by the wave and by the total energy flux (electromagnetic + particles); it involves an important Lorentz in-

W. Sieber and R. Wielebinski (eds.), Pulsars, 53–56.

variant parameter

$$q \cong \frac{8}{\pi\nu_1}\left(\frac{\beta^2}{\beta^2-1}\right)^{1/2} (\gamma_1 - U_1^r/\beta)$$

(with γ_1 the Lorentz factor, U_1^r the radial 4-velocity at maximum strength of the wave ν_1). $q >> 1$ describes sinusoidal waves in the linear approximation with $(\beta^2/(\beta^2-1)) = (\omega^2/W_p^2)$. $q << 1$ describes non-linear l.a.w. with $(\beta^2/(\beta^2-1)) = (2/\pi)^2\ (\omega^4\nu_1^2/W_p^4)$ (W_p^2 is the averaged plasma frequency) and allows propagation of l.a.w. even in very dense plasmas. Such intense waves determine the mean properties of the plasma: the relativistic particles are swept along with mean velocity c/β; there is equipartition between electromagnetic and particle fluxes, thus particles carry away an important part of the rotational energy lost by the neutron star. Propagation of the l.a.w. is effective only when the transported flux J is less than the critical particle flux J* given by Goldreich and Julian (1969) beyond which MHD equations are valid. When J > J*, a relativistic wind solution has to be investigated. If the particle flux deduced from observations in the Crab nebula (Shklovsky, 1970) is supposed to be entirely supplied by the Crab pulsar, a relativistic wind might exist, the magnetospheric plasma being too dense to transmit the l.a.w. (Asséo et al., 1975).

When the amplitude of the l.a.w. varies, its phase velocity and wavelength λ are modified. Consequently the conditions for its propagation are changed. Two kinds of processes, giving such an effect, were investigated: the radiation reaction of relativistic particles moving in and interacting with these waves (Asséo et al., 1978) and also the inhomogeneities induced by spherical geometry more appropriate to the pulsar problem (Asséo et al., 1981). For this purpose we use a WKB type method with two characteristic scale lengths: a rapid scale (≅ wavelength) on which the phase of the wave varies; a slow scale, characteristic of the variations of the wave parameters, on which radiative or spherical geometry effects occur. If radiative or sphericity effects are considered as perturbations, the l.a.w. conserves its characteristics as just described for the plane, homogeneous, non-radiative case. $\Lambda = 2(J^*/J)^3\ (10^{41}/\mathring{W}\omega)\lambda$ is the radiative damping length of the l.a.w. ($\mathring{W}$ is the total energy lost by the pulsar). For the Crab pulsar, $\Lambda \cong 10$ wavelengths (when $J \cong J^*$), and the wave is completely damped. For slower pulsars, the radiative damping is weak but propagation of the l.a.w. may be prevented due to spherical geometry. Inhomogeneities induced by sphericity superimpose periodic components on the sawtooth wave, dependent on the gradients of the physical quantities associated to it, but of very small relative amplitude. This allows us to use the sawtooth wave as a zero order approximation assuming that physical quantities vary slowly with the distance. We obtain the dispersion relation in terms of the measurable quantities, namely the averaged particle, energy and radial momentum fluxes, respectively F_p, F_e and F_r. For the wave to propagate as a l.a.w. in the nonlinear regime, the parameter q has to keep its small value. But $q \cong (16/\pi^2)\ \cdot$

$(F_r/\beta F_p^2)\,\lambda^2$ increases with distance. If the pulsar injects a l.a.w. at λ_o, q increases during the propagation while the amplitude of the wave decreases. Thus inhomogeneities induced by sphericity prevent the wave to propagate as a large amplitude sawtooth wave.

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DISCUSSION

MANCHESTER: Can you estimate the distance scale for the decay of the non-linearity?

ASSEO: With the physical parameters of the Crab pulsar, we estimate the distance for the decay of the l.a.w. to be of the order of 10 wavelengths due to radiation reaction of particles and of the order of 10^2 times the radius of injection due to sphericity effects.

MAX: You have shown that linearly polarized strong waves can only propagate with a certain well-defined mass outflow, and that in spherical propagation they cannot travel very far. By contrast, circularly polarized strong waves are well known to be able to propagate with a much broader range of mass outflow. Do you have any opinion about which is the more general case, i.e. is elliptical polarization more similar to circular, or to linear? Since magnetic dipole radiation has circular polarization over the pole and linear around the equator, this difference in the propagation properties of the two polarizations might produce observable asymmetries in the region surrounding a neutron star emitting dipole radiation.

ASSEO: I agree that for circularly polarized waves there is no unique relation between the cosmic-ray flux transported by the wave and the wave amplitude: no unique statement can be made for such waves. For elliptically polarized waves of relativistic amplitude Kennel predicts that, contrary to intuition, in zero-average magnetic fields particles and Poynting flux are still almost equal and that the wave ceases to propagate if the particle flux exceeds J*.

Circular polarization over the poles is insignificant and no characteristic asymmetry should be expected from the difference between linear and circular polarization.

KUNDT: Where does the 4-momentum of the strong wave go if it cannot propagate?

ASSEO: Radiation reaction dissipates energy into a photon background, which does not interact further with the wave. Particles of the electron-positron plasma, accelerated by the electromagnetic field of the l.a.w. all radiate in a similar way. They produce a coherent photon flux travelling at an angle to the direction of the l.a.w., which does not remain coherent with the l.a.w. Sphericity effects may provide a similar dissipation mechanism, but we have not yet investigated this case.