SIMPLE FORMULA FOR RADIO JET SURFACE BRIGHTNESS

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Henriksen, Bridle and Chan (1981; HBC) have proposed that the energy for the synchrotron emission of radio jets is derived ultimately from the turbulent hydrodynamic eddy cascade between the Taylor wave number, k_T, and the Kolmogorov wave number. This cascade is established during the development of a turbulent jet by the shearing action of large-scale vortical entrainment of ambient material (Brown & Roshko, 1974).

Their formula for the surface brightness, $B(\nu)$, of an optically thin radio jet is

$$B(\nu) = \sqrt{1 - \bar{\omega}^2/R^2} \quad e(\nu) \; A \; (Vj/R)^2 \; (dR/dz)^3 \tag{1}$$

where $R(z)$ is the 'radius' of the jet, Vj is the jet velocity, A is the mass flux in the jet (a function of R when the ambient density profile is sufficiently flat), $\bar{\omega}$ is the cylindrical distance from the jet axis (z) of the measurement, and $e(\nu)$ allows for the spectral distribution of the power in the cascade as

$$e(\nu) = (1 - \alpha) \; / \; (4\pi\nu_{max}) \; (\nu/\nu_{max})^{-\alpha} \tag{2}$$

Either the observed spectral index may be used for α, or the theoretical value of 0.75 (HBC and below) may be used.

The formula needs only $Vj(z)$ in order that the 'shape' $B_\nu(z)$ may be compared to the observation. For NGC 315 this is taken from the CH model fit to $R(z)$ published by Bridle, Chan and Henriksen (1981). The fit to the brightness data of Willis et al. (1981) is shown in Fig.1. The smooth curve is equation 1. The two sections correspond to the two regimes of expansion in NGC 315.

The self-consistent spectral index, α, is a matter for careful calculation (Eilek and Henriksen, 1982). However, HBC have shown that $\alpha = 0.75$, if their 'Lighthill' rate of resonant Alfven wave emission just balances the synchrotron losses in each energy interval. Moreover, the 'Lighthill' wave driving rate given by HBC as

$$I_a(k) \simeq \rho \left((\Delta v)^3 / R\right) \left(\Delta v / v_a\right)^{3/2} \; k_T^{1/2} \; k^{-3/2}, \tag{3}$$

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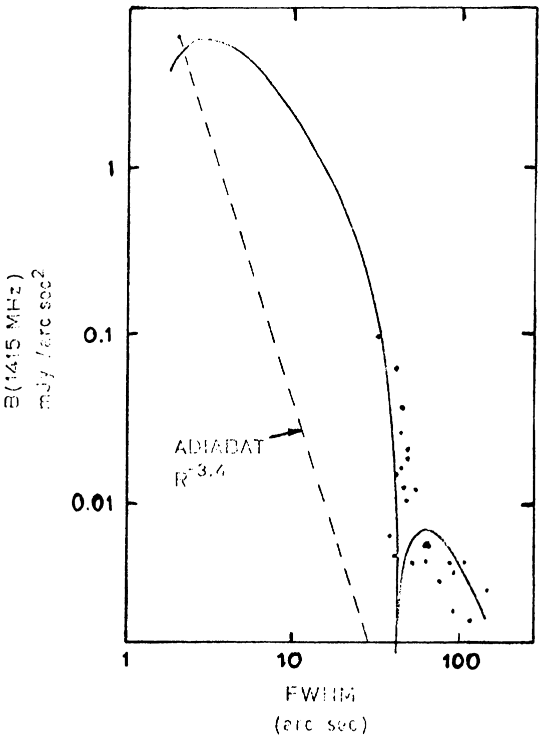

(with $\Delta v \simeq V_j(dR/dz)$, v_a the mean Alfven speed) may be shown (e.g. Eilek, 1979) to yield a steady MHD wave spectrum with energy density $W_a \propto k^{-3}$, provided that the particle spectrum is $dN \propto E^{-2.5}\,dE$. But when $W_a \propto k^{-3}$, Lacombe (1979) has shown that there is a $E^{-2.5}$ self-similar solution for the particle distribution with wave acceleration and synchrotron losses, of the form

$$dN = 2N_T \pi^{-1/2} x^{-2.5} e^{-1/x} Gt\; dE/c \quad (4)$$

where $x \equiv Gt\,E/c$; provided that the ratio of the acceleration time to the loss time is $t_A/t_S = 4.5$. Thus our scheme is self-consistent (ignoring back reaction on the hydrodynamic turbulence) if some reason may be found for the special value of t_A/t_S.

In fact, we have from HBC and Lacombe (1979) that when $W_a \propto k^{-3}$ then (γ_1 is the low energy cut-off of the relativistic particles)

$$\left.\begin{aligned} t_A &= 1/Gp\ ,\quad t_S = 1/Sp \\ G &\simeq (4\pi/3)\,(c/eB^3)\,(\Omega_e k_T/c\gamma_1)^{1/2}\ .\ \rho\ .\ \left((\Delta v)^3/R\right)\,(\Delta v/v_a)^{3/2} \\ S &= (4/9)\ B^2 e^4/(M_e{}^4 c^6) \end{aligned}\right\} (5)$$

Typical values for the main jet in NCG 315 show that indeed $t_A/t_S = 0(1)$. Because t_A/t_S varies with energy when W_a does not vary as k^{-3}, no other spectral shape is stable in time. In fact there is a tendency for t_A/t_S to return to its self-similar value (Eilek and Henriksen, 1982), because of the dependence of t_A on the spectral shape.

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