durch die üblichen Schlußregeln (Einsetzung, und *modus ponens*, d. h. Abtrennung) beweisbar sind; die Vollständigkeit im schärferen Post'schen Sinne wird daraus in einigen Zeilen gefolgert.

Die Methode des Vollständigkeitsbeweises ist eine Wertungsmethode, d. h. von den tautologischen Formeln wird unmittelbar die Eigenschaft ausgenützt, daß sie an jeder Stelle wahr sind. Eine solche Methode wurde zuerst von László Kalmár (3849) zu einem Vollständigkeitsbeweis angewandt. Der Grundgedanke des Kalmár'schen Beweises ist folgender: Es sei χ eine Formel mit den Variablen $p_1, p_2, \cdots, p_r$, die an der Stelle $W_1, W_2, \cdots, W_r$ wahr ist, ferner bedeute $\mathfrak{P}_i$ $(i = 1, 2, \cdots, r)$ p_i oder $\sim p_i$ je nachdem W_i T oder F ist. Nimmt man $\mathfrak{P}_1, \mathfrak{P}_2, \cdots, \mathfrak{P}_r$ als Prämissen zu den Axiomen hinzu, so wird gezeigt, daß χ beweisbar wird. Ist χ eine tautologische Formel, so kann $\mathfrak{P}_i$ beliebig p_i oder $\sim p_i$ gewählt werden, infolgedessen werden die Prämissen eliminiert.

Den hier auftretenden Hilfsbegriff "Beweisbarkeit aus Prämissen" hat bereits M. Wajsberg (II 93(2)) umgegangen, indem er beweist, daß eine Formel χ mit der Gesamtheit gewisser aus χ durch Einsetzungen entstehenden Formeln (von kleinerer Variablenzahl) deduktionsgleich ist.

Quine bildet diese—von Wajsberg auf den Teilbereich der reinen Implikationsformeln verwendete—Methode weiter aus, indem er die Verminderung der Variablenzahl durch Einsetzung der Wahrheitswerte selbst bewirkt. Entstehen ω bzw. ψ aus der Formel χ durch Einsetzung von T bzw. F für eine Variable, oder allgemeiner für eine Teilformel ϕ, so zeigt Quine, daß

$$F)\qquad \phi \supset (\chi \supset \omega) \text{ und } \phi \supset (\omega \supset \chi), \quad \sim\phi \supset (\chi \supset \psi) \text{ und } \sim\phi \supset (\psi \supset \chi)$$

und infolgedessen auch

$$G)\qquad \psi \supset (\omega \supset \chi)$$

beweisbar ist. Aus den Formeln *F*) folgt auch, daß durch Ersetzung von $\mathrm{T} \supset \mathrm{T}$, $\mathrm{T} \supset \mathrm{F}$, $\mathrm{F} \supset \mathrm{T}$, $\mathrm{F} \supset \mathrm{F}$ der Reihe nach durch T, F, T, T in einer Formel, eine deduktionsgleiche Formel entsteht. Daraus ergibt sich ohne weiteres die Beweisbarkeit der tautologischen Formeln ohne Variablen, folglich mit Hilfe von *G*) durch Induktion nach der Anzahl der Variablen die Beweisbarkeit aller tautologischen Formeln. RÓZSA PÉTER

HANS HERMES. ***Eine Axiomatisierung der allgemeinen Mechanik.*** Forschungen zur Logik und zur Grundlegung der exakten Wissenschaften, new series, vol. 3. S. Hirzel, Leipzig 1938, 48 pp.

This monograph gives a set of axioms for the foundation of special relativity, with electrical theory omitted. The two undefined physical terms are "*Gid*", a relation such that *a Gid b* if and only if *a* and *b* are on the same world line, and "*Bzs*", the class of all admissible (inertial) coordinate systems.

The axioms and definitions are expressed in the language of PM. The precision which this would make possible is lost because of the use of a number of mathematical ideas, some of which are explained only very vaguely. For instance, "*lo*" is described as the set of Lorentz transformations of four-dimensional vector space. With no further explanation than this, one would think that the members of "*lo*" would carry the entire four-space into itself. However, on this basis one could deduce from axiom A4.5 that there are no isolated bodies. This would contradict A8.1. The exact change in the meaning of "*lo*" necessary to avoid this contradiction is left to the reader.

The paper is further marred by the presence of a large number of minor errors. Some can be attributed to careless printing or proof reading, such as the following: in axiom A4.5, Σ_2 should be $\breve{\Sigma}_2$; on p. 10 the interpretation of $g(y)$ should be changed to read "$f(x) \rightarrow g(x)$", as otherwise axiom A4.3 would read $\Sigma \epsilon Bzs \;\&\; \Sigma \epsilon 1 \rightarrow 1$. Other errors are of a more essential character, such as the absence of any axiom from which one could deduce the existence of admissible coordinate systems.

The axiom set, when suitably corrected, is sufficient for special relativity, but not necessary. That is, a system which satisfies Hermes's axioms would be a system of special relativity, but not every system of special relativity would satisfy Hermes's axioms. Hence Hermes's axioms do not properly constitute "an axiomatization of general mechanics."

The difficulty seems to lie mainly in axiom A8.1, which says that the corpuscles of matter behave in certain very peculiar fashions. It is quite possible that, in writing A8.1, Hermes really had in mind some sort of conditional statement to the effect that if the corpuscles behave in certain very peculiar fashions, then certain other things would happen. However, as stated, A8.1 is distinctly not conditional. BARKLEY ROSSER

I. M. BOCHEŃSKI. *Notes historiques sur les propositions modales.* ***Revue des sciences philosophiques et théologiques*** (Paris), vol. 26 (1937), pp. 673–692.

The logic of modal propositions is at best a decadent development. Great confusion arises from the use of "possible" to mean, sometimes, "not impossible," and sometimes, "neither impossible nor necessary." The author proposes to use "contingent" for the latter sense.

Aristotle uses "possible" in the sense of "contingent" and Theophrastus uses it in its strict sense. Herein lies their main point of difference with regard to modal propositions, causing Theophrastus to think he had a new system of logic which corrected the "errors" of Aristotle. Albertus Magnus followed Aristotle; "Pseudo-Scotus" expounded the distinction; Ockham combined the two systems and derived 1000 valid forms.

"Pseudo-Scotus" produced interesting proofs of the propositions, "A false proposition implies any proposition" and "A true proposition is implied by any proposition."

S. K. LANGER

R. FEYS. *Les logiques nouvelles des modalités.* ***Revue néoscolastique de philosophie,*** vol. 40 (1937), pp. 517–553, and vol. 41 (1938), pp. 217–252.

A clear, concise systematization of modern contributions to the study of elementary, abstract, modal logics; a summarization and comparison of (0) the classical true-false logic, e.g., that of ***Principia mathematica,*** (1) logics of the traditional modal concepts of necessity, possibility, etc., e.g., Lewis's logic of "strict implication," (2) "intuitionistic" logics, e.g., Heyting's, (3) many-valued logics, e.g., those of Łukasiewicz. The essay provides a valuable basis and stimulus for further investigations in the field.

I would question one remark. Following Wajsberg, the author states that logics of type (1) can be translated into logics of general propositions or classes. For example, he would say that the analogue of Ap, or better, $A(\phi x)$, "ϕx is necessary," is $(x)\phi x$, "for every x, ϕx." But in a calculus of classes or general propositions there can be strict as opposed to material relations, analogous to strict as opposed to material implication as distinguished by Lewis. In the usual extensional treatment of classes and general propositions, where only material relations are explicitly used, $(x)\phi x$ means merely "for every actual x, ϕx" instead of "for every possible x, ϕx," and is not equivalent to "ϕx is necessary." Only a calculus of classes or general propositions using strict relations would contain analogues of the traditional modal forms. CHARLES A. BAYLIS

L. CHWISTEK and W. HETPER. *New foundation of formal metamathematics.* ***The journal of symbolic logic,*** vol. 3 (1938), pp. 1–36.

The primitive signs comprise just the letter "*c*" and a two-place operator "*". These are combined according to a principle familiar to readers of Łukasiewicz: "*cc", "**ccc", "*c*cc", "**cc*cc", etc. The two signs receive meaning only mediately, through interpretation of certain arbitrary complexes thereof. For example, complexes of the form "******cc**cc*cc*EE*FF*GG*HH" (with the capitals supplanted by any expressions built in turn from "*c*" and "*") are abbreviated "(*EFGH*)[*c*]" and explained as meaning that *H* is the result of substituting *G* for *F* in *E*. Another arbitrary mode of combination of "*c*" and "*" is identified with alternative denial (Sheffer's stroke); and in terms of this the other truth functions are developed in familiar fashion. The syntactical notion of substitution and the logical truth functions are thus gathered into a single system. On this basis further syntactical notions are defined, among them the notions of expression and