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BOOK REVIEWS

DUISTERMAAT, J. J. AND KOLK, J. A. C. *Lie groups* (Universitext, Springer, 2000), viii+344 pp., 3 540 15293 8 (softcover), £27.

There are a number of different flavours of introductions to Lie groups, so let us begin by sketching the contents of this one. There are four substantial chapters: Lie groups and Lie algebras (90 pages); Proper actions (40 pages); Compact Lie groups (70 pages); Representations of compact groups (120 pages).

The first chapter is a fairly standard introduction exploiting modern differential geometry. A novelty is the construction of the simply connected Lie group with a given Lie algebra as a quotient of a Banach Lie group of paths. The second chapter, on proper actions of Lie groups on manifolds, features slice theorems and stratification by orbit types. This is clearly important material that is not often found in introductory treatments. The third chapter then discusses maximal tori, roots, Weyl groups, etc., making much use of the general theory of the previous chapter applied to the adjoint action of the group on itself. The authors admit that this is not quite as efficient as the usual route, but I found it easy to agree with them that it is illuminating. There is no statement of the classification of compact groups in this chapter. I assume the authors thought that the algebraic arguments are rather apart from the general drift of the book, but it seems odd to me not to devote a page or two to it, even if all proofs are omitted. The final chapter on representations includes Peter–Weyl and culminates in the Borel–Weil theorem.

All this provides a different and worthwhile treatment of a subject on which there are a number of excellent books already. Apart from the viewpoint (which I hope is apparent from the discussion of the contents), what distinguishes this book for me is that it is much more condensed and covers a lot of things more deeply than, for example, the standard 'Graduate Texts' by Varadarajan and by Bröcker and Tom Dieck. Just occasionally the authors give a little space to good, careful discussions of basic examples, but these are small breaks from a concentrated development. There are a number of diagrams, some illuminating and some (Fig. 1.11.1 on p. 55, for example) incomprehensible. I suspect at least one of the authors did not work out how to add labels to the figures with whatever computer package he was using. There are admirable collections of historical notes and references at the end of each chapter.

While this book will be very valuable as a reference, I find it hard to imagine any but a very exceptional graduate student reading much of it successfully. I am surprised therefore that it appears in Springer's 'Universitext' series, and the authors' glib statement that the prerequisites for reading the book are 'some rudiments of group theory, the basic aspects of analysis in several real variables, and the elementary concepts of differential geometry' misrepresents a book which, for example, uses de Rham cohomology (including the Künneth formula) and Banach manifolds with little or no explanation.

A curious feature of the book is the continual reference to a second volume. Given the quality of the first volume, I hope this will not prove to be in the great tradition of mythical second volumes, but the very detailed nature of the references to it seem to suggest that it will exist. It seems odd though that no sketch of its proposed contents is given.

Apart from these matters, this looks like a good book with a novel viewpoint that many mathematicians will want on their bookshelf, and it should certainly be in every mathematics library.

T. BAILEY

SHEN KANGSHEN, CROSSLEY, J. N. AND LUN, A. W.-C. *The nine chapters on the mathematical art: companion and commentary* (Oxford University Press and Science Press, Beijing, 1999), xiv+596 pp., 0 19 853936 3 (OUP) and 7 03 006101 2/0.947 (Science Press), £110.

The *Nine chapters* is a classic of Chinese mathematics. It was written no later than 100 BC but is derived from earlier works going back to the eleventh century BC In the third century AD an important commentary on the work was provided by Liu Hui, who is described as 'the earliest notable Chinese mathematician' and who also wrote the text *Sea Island Mathematical Manual*. Several other commentators extended Liu's work, notably Li Chunfeng in the seventh century AD. The present work contains a translation of all this material along with extensive introductions and notes dealing with mathematical, historical and pedagogical matters.

Chapter 0 (Introduction) contains a wide-ranging survey of ancient mathematics both in the east and in the west and deals in particular with the scholarship and research devoted to the *Nine chapters*. Then we have Liu's Preface to his commentary, followed by the nine chapters themselves: field measurement; millet and rice; distribution by proportion; short width; construction consultations; fair levies; excess and deficit; rectangular arrays; right-angled triangles. These are followed by the *Sea Island Mathematical Manual*, which we are told 'is a unique work on mathematical surveying unmatched anywhere by any other similar work of the period.' There is an extensive list of references and the Index seems to be quite comprehensive.

Each of the nine chapters as well as the manual is made up of a series of problems with answers, plus in some cases a statement of the method, and descriptions of general rules; the approach is essentially algorithmic. In the present work the authors introduce each of these chapters and the manual with a summary and a discussion of the nature of the problems along with a comparison of what was being done in China and other parts of the world. A typical problem presentation consists of (i) statement of the problem, its answer and its method, (ii) Liu's comments, (iii) comments from Li and others, (iv) extensive notes from the authors. Different fonts are used for these four parts; this of course serves to highlight individual items, but it often gives the page a bizarre appearance. The notes contain many tables and diagrams which are often lengthy; I think it is regrettable that the authors have chosen to insert them in many instances mid-sentence, causing unnecessary disruption to the flow of the text.

We are told of the painstaking way the authors went about their work, carefully discussing individual words and phrases in an attempt to find exactly the right English words to express the ancient ideas. Although I am unable to read the original, I have no doubt that they have succeeded in producing an admirable translation—it certainly reads well. The notes provided by the authors are helpful, even if one is not always convinced by their conclusions or assertions. I noticed quite a few typographical or editing errors; the corrections are obvious in most cases, but a few might be troublesome: on p. 26 the sentence beginning 'In Table 0.6. . . ' has been corrupted; I do not follow the logical structure of the paragraph beginning 'The modern definition . . . ' on p. 85; at the foot of p. 105 the notation ΔAD is suspect (two places); at the top of p. 109 a command has been wrongly entered, so that $1\{frac12$ appears in place of $1\frac{1}{2}$; there are two tables labelled 10.3 (pp. 551, 559). I believe that the authors have produced a book which is both interesting and fascinating and I am sure that it will be of great value to scholars with an interest in Chinese mathematics and more generally to students of ancient mathematics. The casual reader may find the style of presentation a little off-putting.