Part Two is largely devoted to the other ingredients of the main theorem, the topology of infinite cyclic covers, the properties and construction of bands and their relationship with geometric *ribbons*. The aim is to understand the circumstances in which the infinite cyclic cover of a band is a ribbon and when the technique of *wrapping up* can be applied to a ribbon to get a band. The geometric motivation is kept to the fore, and a number of judicious examples assist the exposition.

The third and final part of the book is a scenic downhill section on the algebraic theory. The emphasis is on the module theory of polynomial rings, which play a key role as the natural coefficient ring for the chain complex of an infinite cyclic cover. The basic homological algebra of mapping cones and mapping tori is developed along with the theory of algebraic bands. If (W, c) is a topological CW band then the chain complex of the universal cover of W is a chain complex band. The discussion of local finiteness requires the extension of scalars to a power series ring, and the end complex is then obtained as a mapping cone of an inclusion. There are some nice examples presented in the discussion of the subtleties of algebraic tameness, essential for the reader who likes to feel solid ground – even non-compact ground – underfoot. The book concludes with the algebraic analogues of ribbons, using bounded algebra (in which modules are graded by a metric space) and bounded topology (in which CW complexes are measured by a cellular map to a metric space).

The book gathers together the main strands of the theory of ends of manifolds from the last thirty years and presents a unified and coherent treatment of them. It also contains authoritative expositions of certain topics in topology such as mapping tori and telescopes, often omitted from textbooks. It is thus simultaneously a research monograph and a useful reference, and can be recommended not only to topologists who want the state of the art in the theory of ends, but also to mathematicians who want to see, in a highly non-trivial way, algebra reflected in the mirror of topology and topology fitted to an algebraic framework built for the purpose.

The book is produced to a high standard by the Cambridge University Press, and the complex typesetting problems are solved in an appealing and error-free manner. Publishers and authors are to be congratulated on the achievement.

N. D. GILBERT

FETTER, H. and GAMBOA DE BUEN, B. *The James Forest* (London Mathematical Society Lecture Note Series Vol. 236, Cambridge University Press, Cambridge, 1997), xi + 255 pp., 0 521 58760 3 (paperback), £27.95 (US$44.95).

A common perception of Banach space theory is that it consists of little more than a string of counterexamples. This is a pity, partly because there are many surprisingly strong theorems about arbitrary Banach spaces, and partly because many of the counterexamples that do exist (and it is undeniable that there are several important ones) have greatly enriched the subject, drawing attention to fascinating and unexpected phenomena and not simply killing off interesting problems.

There are four constructions (at least) that stand out as having an importance that transcends the original problem for which they were constructed. In no particular order they are Tsirelson's space, which introduced the idea of inductive constructions and has led to many examples showing that a general Banach space need not have nice subspaces or non-trivial operators, the Kalton-Peck space, which introduced so-called twisted sums, Gluskin's random finite-dimensional spaces, clever sums of which have distinguished many forms of the approximation property, and James's spaces, which have contributed enormously to our understanding of duality and reflexivity. It is to these last spaces that *The James Forest* is devoted.

The most famous property of James's original space, which we shall call J, is that the natural embedding of J into its double dual has codimension one. It follows immediately that J is not isomorphic to its square $J \oplus J$, and is therefore a counterexample to a problem of Banach. A second space constructed by James, known as his tree space, gave a counterexample to another question of Banach by having a non-separable dual but containing no subspace isomorphic to ℓ_1. (This problem was independently solved by Lindenstrauss and Stegall, using a function-space variant of James's space.)

In their comprehensive book Fetter and Gamboa de Buen give complete solutions to these and other problems and prove a variety of properties of the James spaces. They are keen for the book to be accessible to graduate students and for this reason most of their arguments are given in complete detail.

Occasionally their efforts at accessibility can be counterproductive. In particular, they have a tendency not to distinguish between trivial facts and more substantial ones; sometimes it is easier to be told to prove something yourself than it is to follow somebody else's argument. There are many examples of this, of which here is a small sample. On page 14 they attribute to Singer the result that there exists a space which embeds into its double dual with codimension k. (Such a space is called quasi-reflexive of order k.) Here I shall leave the result as an exercise (given what has been said already). On the next page they attribute to Bessaga and Pełczynski the solution of the first Banach problem mentioned above, quoting from a paper of theirs (published ten years after that of James) the result that, if X and Y are quasi-reflexive of order n and m, then $X \oplus Y$ is quasi-reflexive of order $n+m$. This, it seems, was the missing step needed to show that James's space was indeed not isomorphic to its square. On page 28 they need the fact that finite representability is transitive. This is more or less immediate from the definition, but they refer the reader to a book of Beauzamy. This kind of thing can only confuse a graduate student. (Incidentally, on the previous page they appear to confuse Dvoretzky's theorem with the Dvoretzky-Rogers theorem.)

Despite these faults the book will be a very useful reference to anybody who feels that a James-type space may help them to solve one of their problems. In this it will play a similar role to the book of Casazza and Shura on Tsirelson's space (Lecture Notes in Mathematics Vol. 1363, Springer-Verlag, 1989).

W. T. GOWERS

David, G. and Semmes, S. *Fractured fractals and broken dreams – self-similar geometry through metric and measure* (Oxford Lecture Series in Mathematics and its Applications Vol. 7, Clarendon Press, Oxford, 1997), ix + 212pp., 0 19 850166 8, (hardback) £35.

A great many pages have been written about fractals that satisfy the very strong condition of strict self-similarity, where small regions are directly similar to the whole set. Nevertheless, it is easy to construct fractals where parts of different regions resemble each other in a rather weaker sense. For example, if a self-similar set undergoes considerable distortion or breaking, much of the fine-scale geometry will still remain. This book presents a new and wide ranging theory for analysing sets with different parts at different scales looking "vaguely alike".

A set or space is said to satisfy the BPI "big pieces of itself" condition if, given any pair of balls centred in the set, there are substantial subsets within the balls that look roughly alike in the sense of Lipschitz equivalence. There is a similar definition for the BPI equivalence of two sets. The weaker, but related notion of one set "looking down" on another is also central. The book is devoted to developing and applying these notions. For instance, one basic result is that two BPI sets are BPI equivalent if they contain a pair of subsets that are bi-Lipschitz equivalent.

The book is full of examples and constructions of BPI sets, including familiar fractals, Heisenberg groups, nested cubes constructions and deformations of sets. Thus the theory embraces a far wider range of fractal sets than the now familiar iterated function system framework. The development includes results on weak tangents, measure properties and Lipschitz mappings.