

Part 2

Timing, General Relativity  
and Astrometry

Section C. General Relativity

## Small-eccentricity binary pulsars and relativistic gravity

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**Abstract.** Small-eccentricity binary pulsars with white dwarf companions provide excellent test laboratories for various effects predicted by alternative theories of gravity, in particular tests for the emission of gravitational dipole radiation and the existence of gravitational Stark effects. We will present new limits to these effects. The statistical analysis presented here, for the first time, takes appropriately care of selection effects.

### 1. Introduction

The majority of binary pulsars is found to be in orbit with a white-dwarf companion. Due to the mass transfer in the past, these systems have very small orbital eccentricities and, therefore, neither the relativistic advance of periastron nor the Einstein delay were measured for any of these binary pulsars. In fact, the only post-Keplerian parameters measured with reasonable accuracy for a small-eccentricity binary pulsar are the two Shapiro parameters in case of PSR B1855+09 (Kaspi *et al.* 1994). However, since the orbital period of this system is 12.3 days, the expected gravitational wave damping of the orbital motion is by far too small to be of any importance for timing observations and, consequently, there is no third post-Keplerian parameter which would allow the kind of test conducted in double-neutron-star systems (Damour & Taylor 1992). On the other hand, many alternative theories of gravity, tensor-scalar theories for instance, predict effects that depend strongly on the difference between the gravitational self energy per unit mass ( $\epsilon \equiv E^{\text{grav}}/mc^2$ ) of the two masses of a binary systems (Will 1993; Damour & Esposito-Farèse 1996ab). While this difference in binding energies is comparably small for double-neutron-star systems, it is large in neutron star-white dwarf systems since for a white dwarf  $\epsilon \sim 10^{-4}$  while for a  $1.4M_{\odot}$  neutron star  $\epsilon \approx 0.15$ .

### 2. Gravitational dipole radiation

Unlike general relativity, many alternative theories of gravity predict the presence of all radiative multipoles — monopole and dipole, as well as quadrupole and higher multipoles (Will 1993). For binary systems scalar-tensor theories, for instance, predict a loss of orbital energy which at highest order is dominated by scalar dipole radiation. As a result, the orbital period,  $P_b$ , of a circular binary system should change according to

$$\dot{P}_b^{(\text{dipole})} \simeq -\frac{4\pi^2 G_{\star}}{c^3 P_b} \frac{m_p m_c}{m_p + m_c} (\alpha_p - \alpha_c)^2, \quad (1)$$

where  $m_p$  and  $m_c$  denote the mass of the pulsar and its companion, respectively,  $G_*$  is the ‘bare’ gravitational constant and  $c$  the speed of light. The total scalar charge of each star is proportional to its mass and its ‘effective coupling strength’  $\alpha(\epsilon)$  (Damour & Esposito-Farèse 1996b). For a white dwarf companion  $|\alpha_c| \ll 1$  and thus the expression  $(\alpha_p - \alpha_c)^2$  in equation (1) can be of the order one if the pulsar develops a significant amount of scalar charge. In this case the gravitational wave damping of the orbit is completely dominated by the emission of gravitational dipole radiation.

PSR J1012+5307 is a 5.3 ms pulsar in a 14.5 h circular orbit with a low mass white-dwarf companion. Since its discovery in 1993 (Nicastro *et al.* 1995) this pulsar has been timed on a regular basis using the Jodrell Bank 76-m and the Effelsberg 100-m radiotelescope, sometimes achieving a timing accuracy of 500 ns after just 10 min of integration (Lange *et al.*, this conference). In addition, the white-dwarf companion appears to be relatively bright ( $V = 19.6$ ) and shows strong Balmer absorption lines. Based on white dwarf model calculations, a companion mass of  $m_c = 0.16 \pm 0.02$  and a distance of  $840 \pm 90$  pc was derived (van Kerkwijk *et al.* 1996, Callanan *et al.* 1998). Further, a reliable radial velocity curve for the white dwarf has been extracted, which then, in combination with the pulsar timing information, gave a mass for the pulsar of  $m_p = 1.64 \pm 0.22$ . Since  $\dot{P}_b = (0.1 \pm 1.8) \times 10^{-13}$  for this binary system, we find from equation (1)

$$|\alpha_p| < 0.02 \quad (95\% \text{ C.L.}) \quad (2)$$

Simulations show, that this value should improve by a factor of five within the next three years (Lange *et al.*, in prep.).

### 3. Gravitational Stark effects

#### 3.1. Violation of the strong equivalence principle

The *strong equivalence principle (SEP)* requires the universality of free fall of all objects in an external gravitational field regardless of their mass, composition and fraction of gravitational self-energy. While all metric theories of gravity share the property of universality of free fall of test particles (weak equivalence principle), many of them, which are considered as realistic alternatives to general relativity, predict a violation of the SEP. A violation of the SEP can be understood as an inequality between the gravitational mass,  $m_g$ , and the inertial mass,  $m_i$ , which can be written as function of  $\epsilon$ :

$$m_g/m_i \equiv 1 + \delta(\epsilon) = 1 + \eta\epsilon + \mathcal{O}(\epsilon^2). \quad (3)$$

While the analysis of lunar-laser-ranging data tightly constrains the ‘Nordtvedt parameter’  $\eta$  (Müller *et al.* 1997) it indicates nothing about a violation of the SEP in strong-field regimes, i.e. terms of higher order in  $\epsilon$ , due to the smallness of  $\epsilon$  for solar-system bodies. For neutron stars, however,  $\epsilon \sim 0.15$  and thus binary-pulsars with white-dwarf companions ( $\epsilon \sim 10^{-4}$ ) provide ideal laboratories for testing a violation of the SEP due to nonlinear properties of the gravitational interaction (Damour & Schäfer 1991).

In case of a violation of the SEP the eccentricity vector of a small-eccentricity binary-pulsar system exposed to the external gravitational field of the Galaxy,

$\mathbf{g}$ , is a superposition of a constant vector  $\mathbf{e}_F$  and a vector  $\mathbf{e}_R$  which is turning in the orbital plane with the rate of the relativistic advance of periastron. The ‘induced’ eccentricity  $\mathbf{e}_F$  points into the direction of the projection of the Galactic acceleration onto the orbital plane,  $\mathbf{g}_\perp$ , and  $e_F \propto (\delta_p - \delta_c) P_b^2 g_\perp$ . However, neither the length of  $\mathbf{e}_R$  nor its rotational phase  $\theta$  are known quantities. We therefore have to proceed as follows. Given a certain  $(\delta_p - \delta_c) \simeq \delta_p$ , i.e. a certain  $e_F$  for a given binary pulsar, the observed eccentricity  $e$  sets an upper limit to  $|\theta|$  which is independent of  $e_R$ :  $\sin |\theta| < e/e_F$  for  $e < e_F$  and  $|\theta| \leq \pi$  for  $e \geq e_F$  (Wex 1997). We now have to calculate an upper limit for  $\theta$  for every observed small-eccentricity binary pulsar and compare the result with Monte-Carlo simulations of a large number of (cumulative) distributions for the (uniformly distributed) angle  $\theta$ . This way, by counting the number of simulated distributions which are in agreement with the ‘observed’ limits, one obtains the confidence level with which a certain  $\delta_p$  is excluded. As a safe upper limit for  $|\delta_p|$  we find

$$|\delta_p| < 0.009 \quad (95\% \text{ C.L.}) \quad (4)$$

Note, in order to calculate  $e_F$  for a given binary system, we need also the masses of pulsar and companion and the location and orientation of the binary system in the Galaxy. If there are no restrictions from timing and optical observations, the pulsar masses were assumed to be uniformly distributed in the range  $1.2M_\odot < m_p < 2M_\odot$ , the companion masses were taken from evolutionary scenarios (Tauris & Savonije 1999), and the pulsar distances were estimated using the Taylor-Cordes model assuming a typical error of 25% (Taylor & Cordes 1993). Finally, the orientation of the ascending node in the sky, which is an unobservable parameter for all our binary system, was treated as variable which is uniformly distributed between 0 and  $2\pi$ .

### 3.2. Violation of local Lorentz invariance and conservation laws

If gravity is mediated in part by a long-range vector field or by a second tensor field one expects the global matter distribution in the Universe to select a preferred frame for the gravitational interaction (Will & Nordtvedt 1972). At the post-Newtonian level, gravitational effects associated with such a *violation of the local Lorentz invariance* of gravity are characterized by two theory dependent parameters  $\alpha_1$  and  $\alpha_2$ . If  $\alpha_1$  were different from zero, the eccentricity of a binary system which moves with respect to the global matter distribution in the Universe would suffer a secular change similar to a violation of the SEP. This time,  $|e_F| \propto \alpha_1 |m_p - m_c| P_b^{1/3} w_\perp$  where  $\mathbf{w}$  denotes the velocity of the binary system with respect to the preferred frame, i.e. the cosmic microwave background. Again, we can perform a Monte-Carlo analysis as outlined in the previous section to derive the upper limit

$$|\alpha_1| < 1.2 \times 10^{-4} \quad (95\% \text{ C.L.}) \quad (5)$$

This limit is slightly better than the limit obtained from lunar-laser-ranging data (Müller *et al.* 1996) and, more importantly, also holds for strong gravitational-field effects which could occur in the strong-field regions of neutron stars. Due to its small eccentricity,  $e < 1.7 \times 10^{-6}$  (95% C.L.), and high velocity with respect to the cosmic microwave background ( $w \approx 500$  km/s), PSR J1012+5307 turns

out to be the most important binary system for this kind of analysis (Lange *et al.*, in prep). While for PSR J1012+5307 also the radial velocity of the system is known from spectroscopic observations of the white dwarf companion, for all the other binary pulsars no radial velocity information is available and we have to assume an isotropic probability distribution for the 3-d velocity.

In theories of gravity which violate the local Lorentz invariance and the momentum conservation law, a rotating self-gravitating body will suffer a self-acceleration which is given by  $\mathbf{a}_{\text{self}} = -\frac{1}{3}\alpha_3\epsilon\mathbf{w} \times \boldsymbol{\Omega}$  (Nordtvedt & Will 1972), where  $\alpha_3$  is a theory dependent parameter and  $\boldsymbol{\Omega}$  denotes the rotational velocity of the body. Again, binary pulsars are ideal probes for this kind of self-acceleration effects (Bell & Damour 1996). A careful analysis analogous to the previous analyses (SEP, local Lorentz invariance) gives

$$|\alpha_3| < 1.5 \times 10^{-19} \quad (95\% \text{ C.L.}) \quad (6)$$

Note, the statistical tests for gravitational Stark effects presented here for the first time appropriately take care of selection effects by simulating the whole population, therefore, giving the first reliable limits for  $\delta_p$ ,  $\alpha_1$ , and  $\alpha_3$ .

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