OBSERVATIONS OF THE CELESTIAL EPHEMERIS POLE

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Abstract. Space-geodetic measurement systems are capable of determining: (1) a terrestrial, body-fixed reference frame defined in practice by the stated positions and secular motions of a set of observing stations, (2) a celestial, space-fixed reference frame defined in practice by the stated locations of celestial objects, and (3) the rotation parameters linking these two frames together. Five parameters are conventionally used to specify the orientation of the terrestrial frame with respect to the celestial frame: two nutation parameters, two polar motion parameters, and one spin parameter. The celestial ephemeris pole (CEP) is defined as the north pole of that axis about which the spin parameter (UT1) is measured. The two nutation parameters locate the CEP in the celestial frame, and the two polar motion parameters locate the CEP in the terrestrial frame. By examining the frame transformation matrices, an expression relating the location of the rotation pole to that of the CEP can be derived. In order to compare theoretical predictions with observations, results of models for the effect on the nutations of geophysical excitation processes such as diurnal oceanic current and sea level height variations should not only be given in terms of the location of the CEP (rather than of the rotation pole), but must also account for the resonance effects of the free core nutation.

1. The Celestial Ephemeris Pole and its Relation to the Rotation Pole

The time-dependent components of some position vector $\vec{r_c}(t)$ in a celestial, space-fixed reference frame can be related to its components $\vec{r_t}(t)$ in a terrestrial, body-fixed reference frame through a frame transformation which

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can be written as

$$\vec{r_c}(t) = PNUXY\ \vec{r_t}(t) \tag{1}$$

with the precession P, the nutation N, spin U, and polar motion X and Y matrices being given by (e.g., Sovers and Jacobs, 1994):

$$U = \begin{pmatrix} \cos h & -\sin h & 0 \\ \sin h & \cos h & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad ; \quad X = \begin{pmatrix} \cos x_p & 0 & -\sin x_p \\ 0 & 1 & 0 \\ \sin x_p & 0 & \cos x_p \end{pmatrix}$$

$$Y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos y_p & \sin y_p \\ 0 & -\sin y_p & \cos y_p \end{pmatrix}$$

$$N = \begin{pmatrix} \cos\delta\psi & \cos\epsilon\sin\delta\psi & \sin\epsilon\sin\delta\psi \\ -\cos\epsilon_0\sin\delta\psi & \cos\epsilon_0\cos\epsilon\cos\delta\psi + \sin\epsilon_0\sin\epsilon & \cos\epsilon_0\sin\epsilon\cos\delta\psi - \sin\epsilon_0\cos\epsilon \\ -\sin\epsilon_0\sin\delta\psi & \sin\epsilon_0\cos\epsilon\cos\delta\psi - \cos\epsilon_0\sin\epsilon & \sin\epsilon_0\sin\epsilon\cos\delta\psi + \cos\epsilon_0\cos\epsilon \end{pmatrix}$$

where h is the hour angle of the true equinox of date, $\delta\psi$ is the nutation in longitude, $\delta\epsilon = \epsilon - \epsilon_0$ is the nutation in obliquity with ϵ_0 being the mean obliquity, and x_p and y_p are the reported polar motion parameters with x_p defined to be positive along the Greenwich meridian and y_p positive along 90° W longitude. Note that no expression is given in (1) for the precession matrix P since in this discussion the precession will be treated as a 26,000-year nutation and will be assumed to be included in the nutation parameters. The five Earth rotation parameters linking the celestial and terrestrial reference frames to each other are the two nutation parameters $\delta\psi$ and $\delta\epsilon$, the two polar motion parameters x_p and y_p, and UT1 whose definition in terms of h is given by Aoki et al. (1982). Each of the matrices in (1) represents a rotation about some axis. That axis about which the rotation represented by U is taken defines the celestial ephemeris axis, with the CEP being that point of intersection of the celestial ephemeris axis with the Earth's surface near the north geographic pole. The pole to which the polar motion parameters refer can be determined by first assuming that the inverse N^{-1} of the nutation matrix (assumed to include precession) has been applied to $\vec{r_c}$ so that the components of the position vector $\vec{r_c'} = N^{-1}\vec{r_c}$ are given in a rotated celestial reference frame whose z-axis is aligned with the instantaneous celestial ephemeris axis. Let $\vec{r_c'} = (0, 0, 1)^T$ be a unit vector aligned with the instantaneous celestial ephemeris axis in this rotated celestial frame. In the terrestrial frame, its components are to first order given by $\vec{r_t} = (UXY)^{-1}\vec{r_c'} = (x_p, -y_p, 1)^T$. Thus, the polar motion parameters locate the CEP, and not the rotation pole, in the terrestrial frame. Similarly, it can be shown that the nutation parameters $\sin\epsilon_0\delta\psi$ and $\delta\epsilon$ locate the CEP in the celestial reference frame. The relation between the terrestrial locations of the rotation and celestial ephemeris poles can be derived (Gross, 1992; Brzezinski and Capitaine, 1993) by first applying the inverse N^{-1} of the nutation matrix (assumed to include precession) to $\vec{r_c}$ so that the components of the position vector $\vec{r_c'}(t) = N^{-1}\vec{r_c}(t) = UXY\vec{r_t}(t) = A^T(t)\vec{r_t}(t)$ are given in a rotated celestial reference frame whose z-axis is aligned with the instantaneous celestial ephemeris axis. The transformation matrix $A^T(t) = UXY$ is time-dependent since the orientation of the terrestrial frame with respect to the celestial frame is changing due to the rotation of the solid Earth (to which the terrestrial frame is attached). In fact, the components ω_i of the rotation vector $\vec{\omega}$ describing the rotation of the terrestrial frame with respect to the celestial frame can be shown (e.g., Gross, 1992) to be the three independent components of the anti-symmetric matrix $W(t) = \dot{A}A^T(t)$ where the dot denotes time differentiation. With this definition of $W(t)$, the relation between the terrestrial locations of the rotation and celestial ephemeris poles can be shown to be $m(t) = p(t) - i/\Omega\dot{p}$ where $m = (\omega_1 + i\omega_2)/\Omega$, $p = x_p - iy_p$, and Ω is the Earth's mean angular velocity. In the frequency domain, this relation becomes $m(\sigma_t) = (\Omega + \sigma_t)/\Omega\ p(\sigma_t)$. The difference between the terrestrial locations of the rotation and celestial ephemeris poles is therefore seen to be a function of the frequency σ_t of the motion. For frequencies $\sigma_t \ll \Omega$, the location of the rotation pole coincides (to first order) with that of the CEP. However, at nearly retrograde diurnal frequencies (i.e., at frequencies within the nutation band as viewed from the terrestrial frame), this is no longer the case. Thus, when comparing polar motion observations to theoretical predictions at nearly retrograde diurnal frequencies, it is important to account for the fact that the observed polar motion parameters x_p and y_p specify the location of the CEP, and not the rotation pole.

2. Geophysical Excitation of Nutation

The free core nutation (FCN) is a resonance in the Earth's rotation due to the presence of the liquid outer core. The frequency σ_{FCN} of the FCN is estimated (Mathews et al., 1991) to be -1.0023203 cycles per sidereal day (*cpsd*) as viewed from within a rotating, body-fixed terrestrial reference frame, and is therefore located within the nutation band. Thus, the FCN resonance in the Earth's rotation must be taken into account when studying geophysical excitation of the nutations. Sasao and Wahr (1981) derived a numerical expression suitable for use in studying the effect on the nutations

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of geophysical excitation mechanisms which in the frequency domain and written in terms of the terrestrial location $p(\sigma_t)$ of the CEP becomes (Gross, 1993; Brzezinski, 1994):

$$p(\sigma_t) = \left[2.554\,10^{-4}\frac{\Omega}{\sigma_{FCN}-\sigma_t} + 2.686\,10^{-3}\frac{\Omega}{\sigma_{CW}-\sigma_t}\right]\frac{\Omega c(\sigma_t)}{A\Omega\tau} + \left[6.170\,10^{-4}\frac{\Omega}{\sigma_{FCN}-\sigma_t} + 1.124\frac{\Omega}{\sigma_{CW}-\sigma_t}\right]\frac{h(\sigma_t)}{A\Omega} \quad (2)$$

where $\tau = \Omega^2 a^5/(3GA)$, a is the mean radius of the Earth, G is the universal gravitational constant, A is the Earth's least principal moment of inertia, and σ_{CW} is the frequency of the Chandler wobble. Gross (1993) used (2) to predict the effects on the Earth's rotation of ocean tidal current [the $h(\sigma_t)$ term in (2)] and sea level height variations [the $\Omega c(\sigma_t)$ term in (2)] from the theoretical ocean tidal angular momentum results of Seiler (1991). His results for the nutations are given here in Table 1. The predicted effect on the nutations of the K_1 and P_1 ocean tide heights is seen to be strongly affected by the resonant enhancement of the FCN, and the contribution of ocean tidal currents to exciting the nutations is predicted to be as large as 76 *mas* for the K_1 tide. The development of improved numerical ocean tide models that have assimilated TOPEX/Poseidon tide height measurements should lead in the near future to better predictions of the effects of ocean tides on the nutations.

TABLE 1. Predicted Effect of Ocean Tides on Retrograde Diurnal Polar Motion (Nutation)

Tide	Nutation period (days)	Current amp (*mas*)	Current phase (deg)	Height amp (*mas*)	Height phase (deg)	Total amp (*mas*)	Total phase (deg)
K_1	∞	75.8	274.	8910.	148.	8865.	148.
P_1	182.62	37.4	298.	912.	151.	881.	153.
O_1	13.66	71.2	4234.	45.3	142.	83.1	201.

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