BULL. AUSTRAL. MATH. SOC. 49F10, 53A07, 35J, 35J60, 35J66
VOL. 40 (1989) [491–492]

PERTURBING AWAY SINGULARITIES FROM EXTREMA OF GEOMETRIC VARIATIONAL PROBLEMS

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We consider area-minimising surfaces and energy-minimising maps that possess singularities and the question of whether or not perturbations of their boundaries remove the singularities.

In Chapter 1 we introduce a general class of geometric variational problem and present the two main examples: the extrema of *area* and *energy*.

In Chapter 2 we discuss how the nature of a singular area-minimizing surface depends upon its codimension and note that in high codimension singularities may be stable, that is, persist under arbitrary "small" perturbations of the boundary. In the codimension-1 case we consider examples where singularities either persist or are removed by small perturbations of the boundary and discuss the analysis of these phenomena in terms of the eigenfunctions of the Jacobi field operator of the area functional. Using this information we then prove the main theorem of Chapter 2 on the existence of a large class of perturbations which remove the singularity of an area-minimising hypercone C in R^{n+1}: roughly speaking, our result is that sufficiently small perturbations of $C \cap S^n$ which lie at least slightly more on one side of $C \cap S^n$ than the other always bound smooth area-minimising hypersurfaces. The precise statement is given in Theorem (2.4.1).

In Chapter 3 we consider energy-minimising maps. We first discuss the properties of the singular tangent map

$$\varphi : B^n \sim \{0\} \to S^p(\subset R^{p+1}) : x \mapsto \frac{x}{\|x\|}, p \geqslant n-1,$$

and in particular show that if $p \geqslant n \geqslant 7$ then φ is uniquely energy-minimising. After we derive energy estimates and consider the nature of perturbations which correspond to eigenfunctions of the Jacobi field operator of the energy functional we are then able to prove a theorem on the eixstence of a large class of perturbations of φ which remove the singularity of φ. This result, analogous to the result in Section 2.4, shows us that

Received 6th December 1989. Thesis submitted to the Australian National University, October 1988. Degree approved February 1989. Supervisor Professor L. Simon.

sufficiently small perturbations of $\varphi \mid S^{n-1}$ which lie at least slightly more on one side of $\varphi \mid S^{n-1}$ than the other always bound smooth energy-minimising maps. The precise statement is given in Theorem (3.3.1).

As a direct consequence, since the image of φ lies in the equator of S^p, this shows the existence of smooth energy-minimising maps into non-convex manifolds.

Finally, in Section 3.4 we consider how the nature of φ changes as we perturb the image S^n into an ellipsoid E^n_a and show that Theorem (3.3.1) can be generalised in this case.

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