A MODEL OF THE SOLAR CYCLE DRIVEN BY THE DYNAMO ACTION OF THE GLOBAL CONVECTION IN THE SOLAR CONVECTION ZONE

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Extensive numerical studies of the dynamo equations due to the global convection are presented to simulate the solar cycle and to open the way to study general stellar magnetic cycles. The dynamo equations which represent the longitudinally-averaged magnetohydrodynamical action (mean magnetohydrodynamics) of the global convection under the influence of the rotation in the solar convection zone are considered here as an initial boundary-value problem. The latitudinal and radial structure of the dynamo action consisting of a generation action due to the differential rotation and a regeneration action due to the global convection is parameterized in accordance with the structure of the rotation and of the global convection. This is done especially in such a way as to represent the presence of the two cells of the regneration action in the radial direction in which the action has opposite signs, which is typical of the regeneration action of the global convection. The effects of the dynamics of the global convection (e.g., the effects of the stratification of the physical conditions in the solar convection zone) are presumed to be all included in those parameters used in the model and they are presumed not to alter the results drastically since these effects are only to change the structure of the regeneration action topologically. However, since the structure of the differential rotation is not known precisely, several typical cases of the differential rotation are examined. A nonlinear process is included by assuming that part of the magnetic field energy is dissipated away when magnetic-field strength exceeds some critical value, simulating the formation of active regions and subsequent dissipation. By adjusting the parameters within a reasonable range, oscillatory solutions (the dynamo waves) are obtained to simulate the solar cycle with periods of the right order of magnitude and with patterns of evolution of the latitudinal distribution of the toroidal component of the magnetic field near the surface similar to the observed Butterfly Diagram of sunspots. In those cases, which simulate the solar cycle well, a slight radial gradient of differential rotation increasing inward plays a role, but the latitudinal equatorial acceleration dominates the differential rotation at least near the surface where sunspots are assumed to be formed. The evolution of the latitudinal distribution of the radial (poloidal) component of the magnetic field shows patterns similar to the Butterfly Diagram but having two branches of different polarity in each hemisphere, predicting that the solar general magnetic field has quadropole-like structure. The development of the radial structure of the magnetic field associated with the solar cycle is presented to explain the simultaneous presence of active regions with the polarity of the new cycle in a region of magnetic field with the polarity of the old

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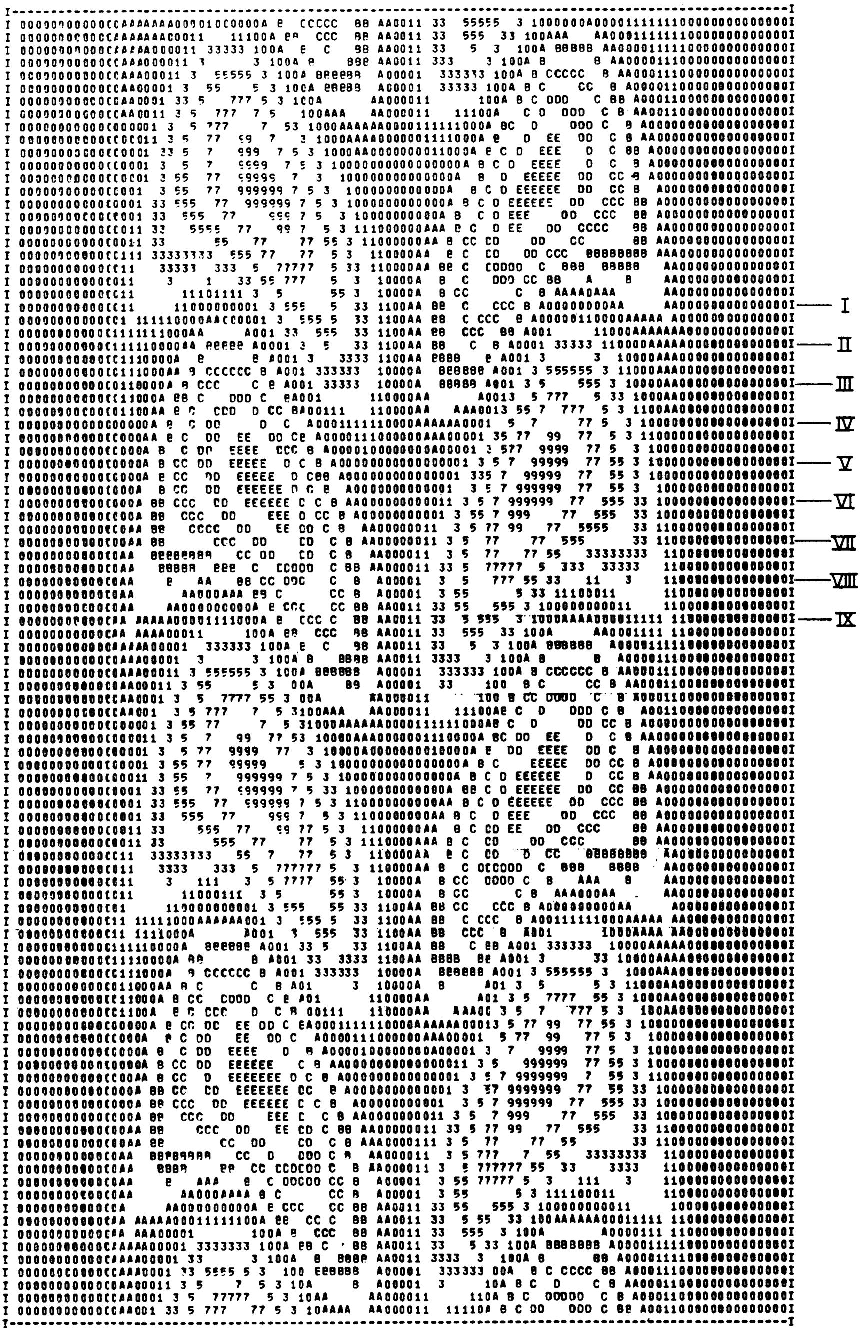

Fig. 1. The evolution of the latitudinal distribution of the *toroidal* general magnetic field near the surface for the standard case of this study in which the latitudinal gradient of the differential rotation (equatorial acceleration) plays the dominant role with the presence of slight radial gradient (rotation increasing inwards). Abscissa is sin (latitude) from south to north pole and the ordinate is time evolving from upward to downward. Note that this diagram well represents the observed Butterfly Diagram of sunspots and that there are slight poleward propagating branches.

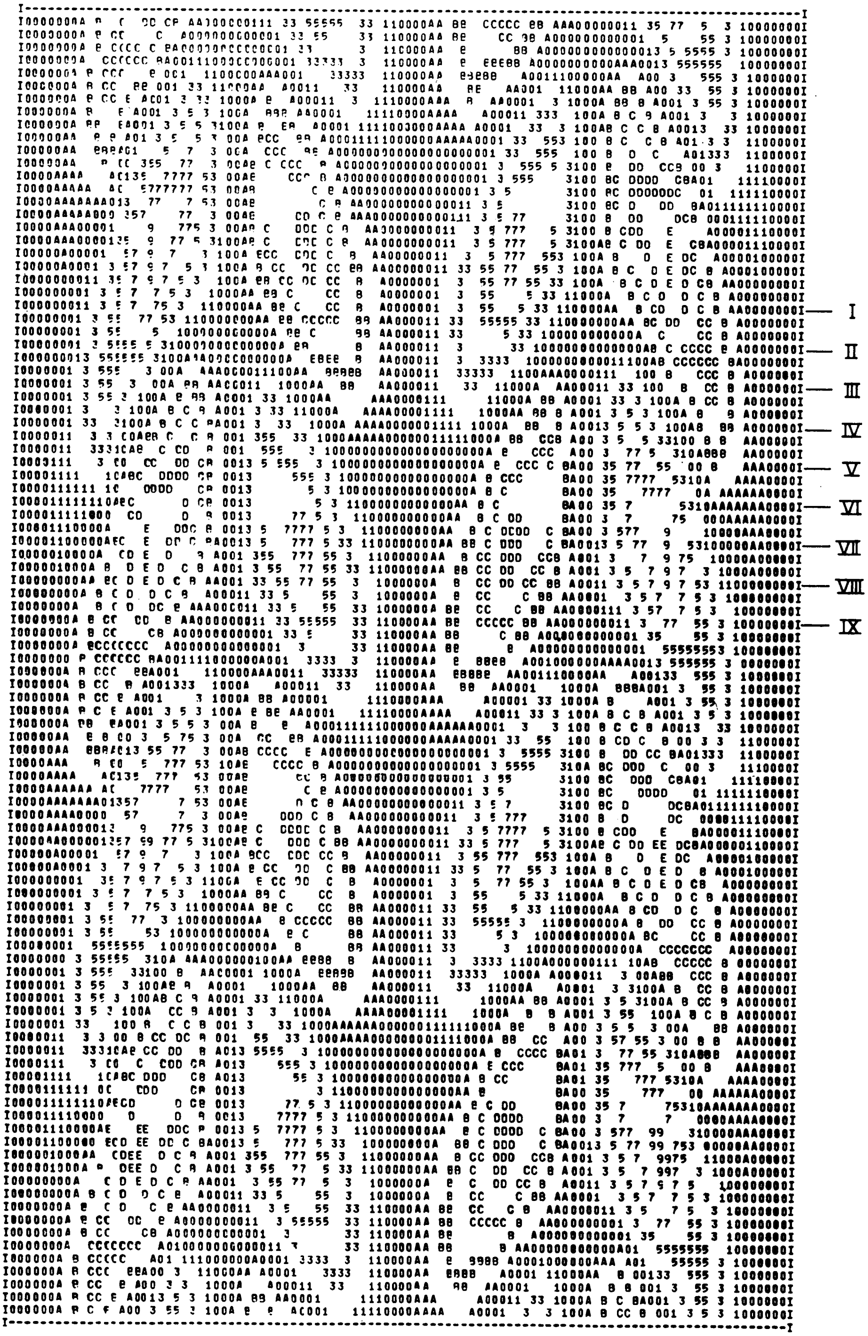

Fig. 2. The same as Figure 1, but of the *poloidal* (*radial*) general magnetic field. Note that there are two branches (wings), in each hemisphere. This predicts that the Sun behaves quadrupole-like not dipole-like. This prediction has been confirmed by Yoshimura (1975c, 1976a, b). This branching is due to the fact that the two (northern and southern) hemispheres behave rather independently. The branching is sensitive to the rotational law of the convection (dynamo) zone of the Sun.

cycle. The importance of the poleward migrating branch of the Butterfly Diagrams of the toroidal and poloidal fields, barely seen for sunspots but clearly seen for polar prominences and for the observed general magnetic field, is emphasized in relation with the relative importance of the role of the latitudinal and radial shears of the differential rotation. Furthermore, by studying general cases of the differential rotational law, the evolution of the magnetic field inside the Sun is found to show that the dynamo waves of the magnetic field propagate along isorotation surfaces (this was done by making a movie showing the evolution of the field); e.g., in the case of purely latitudinal differential rotation, the waves propagate radially, while in the case of purely radial differential rotation, they propagate latitudinally. The direction of the propagation and the phase shift between the evolutions of the poloidal and toroidal fields depend upon the sign of the regeneration action as well as on the rotational law. The importance of the explicit recognition of these results, which hold for any law of the differential rotation, is stressed with regard to the problem of inferring the actual rotational law inside the Sun and stars in general. Further, the general dependence of the characteristics of the dynamo waves responsible for the solar cycle on those parameters adopted, i.e., on the structure of the generation and regeneration actions, is discussed in order to understand the nature of the solar cycle and to parameterize the basic characteristics of the solar cycle. The present formulation of the dynamo equations can be applied to further studies of the solar and stellar cycles.

References

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DISCUSSION

Gilman: If I understand correctly, you included one particular differential rotation profile in your calculation of regeneration action produced by convection, but then you used this regeneration action form for dynamo calculations with other differential rotation profiles. This seems inconsistent to me, if the regeneration action is sensitive to the presence of differential rotation.

Yoshimura: I think you misunderstand. I did not assume one particular form of the differential rotation in my calculation of the regeneration action of the dynamo processes. In my model of the solar cycle, I adopted and tested various forms of the regeneration action with general properties of the action which are not influenced much by the form of the differential rotation as far as it is equatorial acceleration. I parameterized the general properties so that the regeneration action changes its form only topologically and searched for the correct forms of the regeneration action and of the differential rotation which well explain the observed characteristics of the solar cycle. The exact consistency of the two forms should be examined by studying the dynamics of the convection zone. However, this is not the present approach. My approach is opposite to this; i.e., the purpose is to determine the forms of the regeneration action and of the differential rotation, which are useful when we study the dynamics of the convection zone, by comparing the solutions of the model with the observed characteristics of the solar cycle. The flexibility of the formulation of the present model, in my opinion, can well serve for this purpose.

Gilman: Please summarize the condition for the last case in the movie, in which propagation of the dynamo wave is toward the equator.

Yoshimura: In the last case, the radial gradient of the differential rotation predominates the process. And the rotation increases inwards. The structure of the regeneration action is such that the upper part of the action predominates the process, which means the action with positive (negative) sign in the northern (southern) hemisphere predominates the dynamo process. Then the dynamo waves, the amplitude of which is considerable only in the upper part, propagates along isorotation surfaces which are parallel to the surface toward the equator.

Durney: Does your model rule out an angular velocity decreasing with depth in the lower part of the convection zone?

Yoshimura: No, not necessarily, if the radial gradient decreasing inwards does not predominate the process. In order to answer this question correctly, there are many problems to be solved. If the active regions originate in the upper part of the convection zone, the Butterfly Diagram of sunspots should not reflect the distribution of the toroidal field of the lower part. In this case, the form of the differential rotation in the lower part could not be determined by observing the surface phenomena as far as the differential rotation in the lower part does not affect the field in the upper part so much. If the active regions originate in the lower part, the phenomena observed at the surface would greatly be affected by the form of the differential rotation in the lower part which directly affects the field in the lower part. In this case, the angular velocity in the lower part may be decreasing inwards since this brings about the equatorial propagation of the dynamo waves in the lower part. However, to make a comment on Dr Durney's question, I think he has in mind that the iso-rotation surfaces of the differential rotation may be cylindrical, which means that the rotation rate is decreasing inwards if we fit it to the equatorial acceleration observed at the surface. I tested this case using my model of the solar cycle and found that it is difficult to explain the various observed characteristics of the solar cycle by this case of cylindrical form of the differential rotation which has been predicted by some investigators of the differential rotation.

Roxburgh: What would happen if you had many cells in the radial direction as I would expect in a compressible medium. Have you done or is it possible to do calculation for a multi-cell model?

Yoshimura: Do you mean a multi-cell structure in the radial direction with global horizontal scale? If so, I think the model is quite unlikely because we have already difficulties to explain the elongated form of the global convection of one cell. Moreover, various observed surface phenomena of magnetic field and activity can well be explained by one-cell model of the global convection. To answer this question correctly, we should study the dynamics of the convection zone. At present, we have no correct answer. Even for the case that the global convection has a multi-cell structure, it is possible to solve the dynamo equations of my model of the solar cycle if the number of cell is not so large. However, I have not done it yet.

Stix: If there are a number of large scale circulation cells above each other in the convection zone, it appears to me that a meaningful regeneration term can only be obtained by averaging vertically over a number of them, in addition to the longitudinal averaging employed by Dr Yoshimura. But then the Boussinesq approximation would yield no regeneration. We would need the density stratification, as has been emphasized by Dr Krause and Dr Rädler in their papers.

(Note added in proof by H. Yoshimura): If we average the regeneration term vertically, we should also average the original MHD equations vertically. Then the resulting equations have only one space coordinate, i.e., the latitude, as in the case of Leighton's model of the solar cycle. In this case, we have some difficulties if the vertical space to be averaged is comparable to the depth of the convection zone. For, in this case, the differential rotation would not depend on the vertical coordinate. Moreover, the latitudinal gradient of the rotation rate would bring about the vertical propagation of the dynamo waves, the phenomenon which cannot be described by one space coordinate model. Only in the case that the vertical space to be averaged is much smaller than the depth of the system, various physical quantities (as well as the dynamo equations) can have vertical structure which is essential to describe the behavior of the dynamo waves. But, in order to assume this, the fluid motions which are responsible for the dynamo processes, should have much smaller scale than that of the system. In the case of the global convection, this seems to be very unlikely even though the global convection has a multi-cell structure in the radial direction.